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IMCA S 015

Guidelines for GNSS positioning in the oil & gas industry

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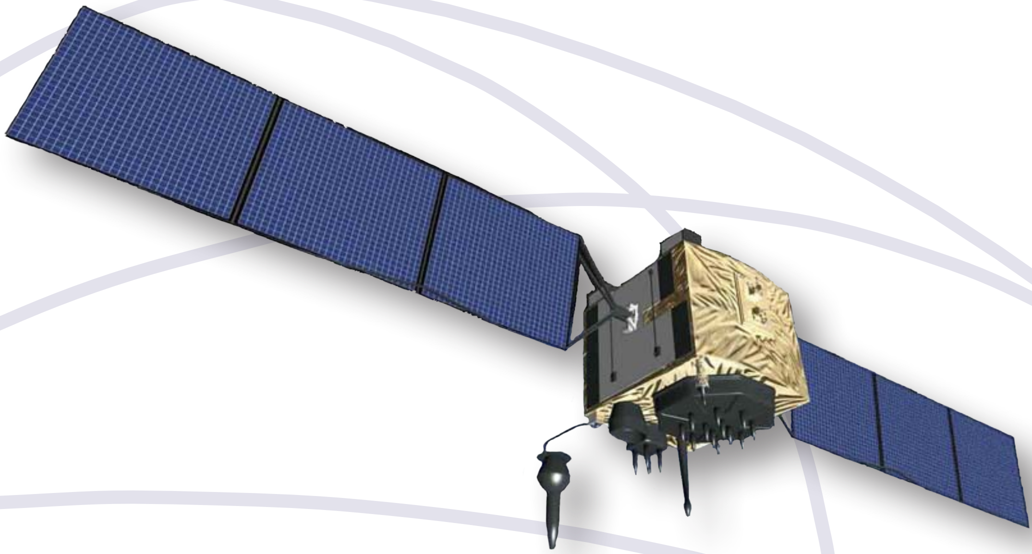

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International Marine Contractors Association

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Geomatics Guidance note 19
IMCA S 015

Guidelines for GNSS positioning in the oil & gas industry

Revision history

Version	Date	Amendments
1	June 2011	Initial publication

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Preface

Since the issue of the *Guidelines for the use of differential GPS in offshore surveying* by the United Kingdom Offshore Operators Association (UKOOA) in 1994, these have been widely adopted by the oil exploration and production industry. In 2005 the International Association of Oil & Gas Producers (OGP) Geomatics Committee (formerly Surveying & Positioning Committee) took over custodianship of the guidelines from Oil & Gas UK (formerly UKOOA). At that time it was generally recognised that the original guidelines needed updating, as they were published when use of differentially corrected GPS for geomatics was still in its infancy, and there had been no revision of the text since that time. A revision of the Guidelines was therefore initiated in order to update the document to reflect current use and developments in GNNS technology and maintain purpose and relevance of the Guidelines. The OGP Geomatics committee agreed to co-operate with the IMCA Offshore Survey Division on the revision of the Guidelines.

A workgroup comprising three members from each organisation was formed in late 2008 and tasked with drawing up a scope and a recommendation for the possible update of the original UKOOA guidelines. Their recommendation to undertake a revision was accepted by both the OGP and IMCA committees and work commenced in April 2009.

The workgroup assumed responsibility for updating the guidelines and, prior to the start, the workgroup was supplemented with further members from seismic and survey contractors. The guidelines were written by the workgroup following discussions and incorporate comment, feedback and suggestions from a range of interested groups within the E&P industry. Comments and suggestions for improvement are welcome and may be addressed to the either the OGP secretariat.

1 Executive Summary

This document, published jointly by the International Association of Oil & Gas Producers (OGP) Geomatics Committee and the International Marine Contractors Association (IMCA) Offshore Survey Division Management Committee, supersedes the previous *Guidelines for the use of differential GPS in offshore surveying* published in 1994 by the Surveying & Positioning Committee of the United Kingdom Offshore Operators Association (UKOOA).

The document provides guidelines for the use of global navigation satellite systems (GNSS) to position vessels, vehicles and other fixed and mobile installations during oil exploration and production (E&P) related surveying and positioning activities. It represents an overview of the recommended principles for reliable positioning and includes recommended minimum statistical testing and quality measures essential for rigorous quality control and performance assessment. Although primarily aimed at E&P related surveying and positioning activities, the principles and recommendations of this guideline are equally valid for any similar activities where precise position is critical, for example: renewable energy plants, cable laying operations, harbour construction and management, etc.

The document assumes a basic knowledge of GNSS. References to text books and other sources are included where relevant and readers are encouraged to refer to these for in-depth descriptions of the principles. The document has been written as far as possible in plain and general terms to provide guidance to those wishing to use GNSS techniques and requiring a suitably authoritative source to support them.

The document does not aim to review the costs of GNSS systems and augmentation services other than to emphasise that each system may have numerous configuration and installation options as well as performance differences. It is recommended that selection of positioning systems and services to provide adequate positioning accuracy and service availability, should not be driven by cost alone.

The main sections of the guidelines are arranged in the general sequence that would occur during the positioning task.

- **Section 2: Abbreviations, acronyms and definitions**
- **Section 3: Introduction and Background** describes the various forms of positioning, available observables and augmentations and the methods and techniques used to create the positioning solution. It outlines the main sources of suitable positioning services and factors that should be considered when selecting positioning systems. The section provides considerations for good practice in the design and planning of positioning operations. The guidelines are primarily aimed at applications supporting marine E&P activities but also cover techniques and applications used on land. However, some specific land survey techniques may not be covered and some land methodologies may be only briefly described.
- **Section 4: Installation and Operation** outlines the preparatory work, geodetic aspects and the various considerations relating to the installation, set up and commissioning of satellite-based positioning systems. Once systems are fully operational there are a number of factors that may affect positioning performance. Within the limits of these guidelines it has not been possible to describe all the many aspects and influences on system installations and their subsequent performance. However some general items are included to alert the user and to illustrate the types of challenges and factors impacting the installation and operation of GNSS equipment.
- **Section 5: Quality Measures** describes the types of errors affecting GNSS based positioning, and their detection and mitigation while remaining operational and within the desired performance levels. The underlying statistics and quality assessment methods are described in order to provide a general understanding of the methods used to identify, isolate and remove errors from the positioning calculation. This key section does not provide an in-depth discourse on statistics and numerical testing; however, it is supported by Appendix A, Estimation and Quality Control, and the reader may refer to several reference texts if necessary.
- **Section 6: Competences** provides guidance in relation to the competences considered necessary for the personnel that operate satellite-based positioning systems.
- **Section 7: Data Formats** briefly describes GNSS receiver data output formats and some commonly used positioning data exchange formats.
- **Appendix A** provides a mathematically description including relevant derivations and examples of position estimation and quality control.

Table 1 lists the recommended statistics and measures for assessing the quality of GNSS position fixes. The method by which the quality measures should be implemented into processing procedures is shown in Figure 12. The techniques are equally applicable to differential, kinematic and clock and orbit corrected GNSS augmentation techniques. However, the exact configuration of the user's mobile equipment may limit the access and monitoring of the quality measures. These guidelines aim to promote good practice in assessing satellite positioning performance and the use of agreed quality measures is a key element in generating reliable positioning.

Measures	**Effect**	**Recommended Value**
Level of significance (α_0)	• Probability of rejecting a valid observation • Size of internal and external reliability measures	1%
Detection power ($\gamma_0 = 1 - \beta_0$)	• Probability of rejecting an invalid observation • Size of internal and external reliability measures	80%
F- test	Acceptance or rejection criterion (unit variance) for full functional and stochastic model	n/a
Critical value w-test	Acceptance or rejection criterion for a single observation	2.576 (99%)
Multiplication factor, 1D	Scale standard error ellipse to desired confidence region	1.96 (95% region)
Multiplication factor, 2D	Scale standard error ellipse to desired confidence region	2.448 (95% region)
Multiplication factor, 3D	Scale standard error ellipsoid to desired confidence region	2.796 (95% region)
Ratio major and minor axis	Isotropy of 2D solution	< 2*
Marginally detectable error (MDE)	Effect on 3D position of the minimum error that can just be detected in an observation with a given level of significance and detection power	n/a

* under normal operating conditions, dependent upon geographic location

Table 1 – Summary of recommended parameters for assessing the quality of GNSS position fixes

The key recommendation of this guideline is that GNSS based positioning should be based on the least squares adjustment principle. In order to carry out rigorous QC, the covariance matrix and residuals generated by the least squares computation should be used to generate **test statistics** and **quality measures.** These quality measures and their usage is based upon the so-called "Delft method" of quality assessment. The recommended test statistics are the **w-test** used to detect outliers and the **F-test** (unit variance test) used to verify the model.

Test Statistics:

- w-test used to detect outliers;
- F-test (unit variance test) used to verify the model which is being used to account for 'errors'.

The quality measures which should be computed and examined for each fix are the **error ellipse** and **external reliability** (3D positional marginally detectable error).

Quality Measures:

- error ellipse;
- external reliability.

Further, when quoting an accuracy or other quality figure, it is always necessary to supply an associated confidence level. In this document figures have been quoted at the two sigma (95%) confidence level, unless indicated otherwise.

2 Glossary

The following terms and acronyms are used throughout this publication and are defined here for clarity. The use of italics in the definition column refers to another term in the Glossary.
Additional text providing clarification of the definition or an example are shown in smaller font.
The source of the definition is indicated in italics and square brackets where relevant and when the source is used several times. The following sources are used:
[EPSG]: Definition from EPSG Geodetic Parameter Dataset. See http://info.ogp.org.uk/geodesy/
[ISO/TC211]: Definition from ISO/TC211 website or the 'ISO/TC211 Multi-Lingual glossary of terms': http://www.isotc211.org/TC211_Multi-Lingual_Glossary-2010-06-06_Published.xls
[OGP]: Definition from the International Association of Oil & Gas Producers website or from OGP Guidance Note 7, Part 1: *Using the EPSG geodetic parameter dataset*: http://www.epsg.org/guides/docs/G7-1.pdf

Term	Explanation
1 – PPS	A one pulse per second timing output available from many GNSS receivers
3D	Three dimensions comprising two horizontal and the vertical direction
Absolute positioning	Positioning technique using augmentation data from a network of GNSS tracking stations correcting errors in the basic broadcast navigation data (orbit and clock) and employs various earth and atmosphere models
Accuracy	The accuracy of a measurement is its degree of closeness to its actual (true) value. As used herein, accuracy is the combination of the precision and reliability of an observation. See also Precision and Reliability
Antenna offsets	The relative 3D spatial measurements between a GNSS antenna position and some other particular point of reference, e.g. centre of gravity of a ship
Augmentation data	Additional information e.g. from a reference or tracking station, applied at a user receiver to improve the positioning solution. See also Differential GNSS
Carrier phase	The observable derived from the carrier signal transmitted by GNSS satellites. See section 3.1
Code	The code signal transmitted by GNSS satellites. There are several available depending upon the receiver used to receive the signals. See section 3.1
COMPASS	Name given to the Chinese satellite positioning system. Originally developed by the military, it is based upon 35 middle-earth orbit satellites and there will be two levels of services, one military and one civilian. At the time of publishing (2011) the system is still under development.
Co-ordinate Reference System	[ISO/TC211]: coordinate system that is related to an object by a datum. NOTE 1: For geodetic datum and vertical datum|, the object will be the Earth NOTE 2: Coordinate reference system is normally abbreviated to CRS. NOTE 3: Types of CRS distinguished in ISO 19111 are: geodetic CRS, projected CRS, vertical CRS and engineering CRS. In the EPSG Dataset geodetic CRS is sub-divided into geocentric CRS, geographic 3D CRS and geographic 2D CRS).
CRP	Common Reference Point of a vessel of vehicle from which offsets to GNSS antennae are measured. On a survey vessel typically the selected to coincide with the centre of gravity.
CRS	See Co-ordinate Reference System

Term	Explanation
Datum	[ISO/TC211]: parameter or set of parameters that define the position of the origin, the scale, and the orientation of a coordinate system. NOTE: See also geodetic datum, vertical datum and engineering datum.
Differential GNSS (DGNSS)	Augmentation technique requiring a GNSS receiver placed at one, or many, known points from which GNSS observable (pseudo-range) corrections can be deduced. The application of such corrections in a mobile receiver
DOP	Dilution of Precision
Double differencing	Two-part technique that begins with single differences formed by subtracting observation equations from a pair of GNSS receivers observing a single satellite. Taking the difference between the two single differences gives the carrier phase double difference
Dynamic	Other than a contextual meaning, herein means any activity where the GNSS receiver is in motion
E&P	Exploration and Production sectors of the hydrocarbon (oil and gas) industry
EGNOS	The European Geostationary Navigation Overlay Service is a space-based augmentation system (SBAS) using geostationary satellites to transmit differential corrections in Europe. It is owned by the European Commission (EC) and operated by the European Satellite Service provider (ESSP).
Ellipsoid	[ISO/TC211]: surface formed by the rotation of an ellipse about a main axis. NOTE: In ISO 19111 and the EPSG Dataset ellipsoids are always oblate, meaning that the axis of rotation is always the minor axis.
Ellipsoid height	[ISO/TC211]: distance of a point from the ellipsoid measured along the perpendicular from the ellipsoid to this point, positive if upwards or outside of the ellipsoid. NOTE 1: Only used as part of a three-dimensional ellipsoidal coordinate system and never on its own. NOTE 2: Ellipsoidal height is commonly designated by h. NOTE 3: See also gravity-related height.
Epoch	An instant of elapsed GNSS time. See also GPS time
Epoch date	The year and Julian day that a specific realisation of the ITRF is applied
EPSG	[OGP]: acronym of the European Petroleum Survey Group, formerly a forum of chief surveyors and geodetic experts from European-based E&P operators. This forum has been absorbed into The International Association of Oil & Gas Producers as the OGP Geomatics Committee. The acronym EPSG remains associated as a brand name with the EPSG Geodetic Parameter Dataset, a product of the original EPSG.
EPSG Geodetic Parameter Dataset	[OGP]: dataset of geodetic data objects with worldwide coverage, published by OGP. NOTE 1: Also known as EPSG Dataset. NOTE 2: The dataset is distributed through a web-based delivery platform [see EPSG Registry], or in a MS Access relational database and SQL script files. See http://info.ogp.org.uk/geodesy/

Term	Explanation
EPSG Registry	[OGP]: the EPSG Geodetic Parameter Registry, a web-based delivery platform for the EPSG Geodetic Parameter Dataset. NOTE: The EPSG Registry can be accessed in any web browser, using URL www.epsg-registry.org.
F-test	One of the quality measures (statistics) used to verify the model which is being used to account for errors in the GNSS solution. See also Quality measures
GAGAN	GPS aided Geo Augmented Navigation is an augmentation system developed jointly by the Indian Space Research Organisation and Airports Authority of India
Galileo	European Union autonomous GNSS system operated by a civilian organisation. At the time of publishing (2011) the system is still under development.
Geoid	An equipotential surface that would coincide with the mean gravity at the sea level surface of the earth. It approximates closely to, but not exactly to, mean sea level
Geocentric CRS	[OGP]: a geodetic CRS using an earth-centred Cartesian 3D coordinate system; the origin of a geocentric CRS is at the centre of mass of the Earth NOTE 1: Also known as ECEF (Eart-Centred, Earth-Fixed) NOTE 2: Associated coordinate tuples consists of X, Y and Z coordinates NOTE 3: Definition from 'OGP Guidance Note 7, Part 1: Using the EPSG Geodetic Parameter Dataset', with URL: http://www.epsg.org/guides/docs/G7-1.pdf
Geodetic CRS	[ISO/TC211]: coordinate reference system based on a geodetic datum. NOTE: See geocentric CRS, geographic 2D CRS, geographic 3D CRS.
Geodetic datum	[ISO/TC211]: datum describing the relationship of a two- or three-dimensional coordinate system to the Earth.
Geoid	[ISO/TC211]: equipotential surface of the Earth's gravity field which is everywhere perpendicular to the direction of gravity and which best fits mean sea level either locally or globally
Geostationary communication satellites	The commonly used constellation of communication satellites lying in the earth's equatorial plane and used for broadcasting augmentation data
GLONASS	Global navigation satellite system. An autonomous satellite positioning system operated by Russian Military Space Forces
GNSS	Global navigation satellite system
GPS	Global Positioning System. An autonomous satellite positioning system operated by the United States Air Force Space Control
GPS Master Control Station	Ground control centre that operates, monitors and maintains the GPS satellite constellation
GPS time	Time count that commenced at 0:00 hours UTC on 6 January 1980. GPS Time is not perturbed by leap seconds therefore runs ahead of UTC
Gravity-related height (or depth)	[ISO/TC211]: height (or depth) dependent on the earth's gravity field NOTE 1: See also ellipsoidal height. NOTE 2: Gravity-related height is normally designated by H, and depth by D.
GTRS	Galileo Terrestrial Reference System

Term	Explanation
Height	See gravity-related height and ellipsoidal height.
Height aiding	Augmentation method using known antenna height as an additional observable
HF	High frequency
IALA	International Association of Lighthouse Authorities
IERS	The International Earth Rotation and Reference System Service. Established in 1987 by the International Astronomical Union and the International Union of Geodesy and Geophysics. The primary objectives of the IERS are to serve the astronomical, geodetic and geophysical communities by providing among other: The International Terrestrial Reference System (see ITRS) and its realisation, the International Terrestrial Reference Frame (see ITRF).
IMCA	The International Marine Contractors Association, www.imca-int.com
IMU	Inertial measurement unit is a device with multi-axis sensors that determines the rate of motion and acceleration
Ionosphere	The ionised region of the atmosphere lying at an altitude between 50 and 1000 km. See section 3.1
ITRF	ITRFyyyy is the International Terrestrial Reference Frame, a specific realisation of the International Terrestrial Reference System (ITRS) consisting of a set of station co-ordinates with velocities for a reference date in the year yyyy, When used in a geodetic context the frame definition is usually supplemented by the epoch specified as a decimal year indicating the reference epoch for which a particular geodetic survey has been executed. This implies that the station co-ordinates as published by IERS have been extrapolated to the specified epoch to account for the effects of tectonic plate motion since the ITRFyyyy definition epoch. See Epoch Date.
ITRS	The International Terrestrial Reference System is a conceptual, idealised dynamic reference system, realised by the International Earth Rotation and Reference System Service (IERS, see same) as a set of co-ordinates and velocities of worldwide geodetic stations. The set of station co-ordinates and velocities is referred to as ITRF. ITRS realisations is calculated at certain epochs. See ITRF.
Ku-band	Communications waveband (12-18 GHz) used primarily for satellite data links
L1, L2, L5	GPS frequencies
Marine radio beacon	Maritime DGNSS service using terrestrial HF radio frequency transmitters to transmit correction signals primarily for coastal navigation. See also IALA.
MF	Medium frequency
MSAS	Multi-functional Satellite Augmentation System. Civilian operated GPS augmentation System covering Japan
MDE	Marginally detectable error
MODU	Mobile offshore drilling unit
Multipath	Radio frequency signals reflected off a surface causing antenna equipment to receive multiple erroneous signals. The bias caused by mixing indirect (reflected) with direct GNSS signals
NGIA	National Geospatial Intelligence Agency (of the USA)

Term	Explanation
NMEA	National Marine Electronics Association (of the USA)
Observables (L1/L2, L5 etc.)	Commonly the GNSS satellites providing signals in space for positioning are equipped with a variety of frequency and signalling options to aid in mitigating interference and offering some benefits. Although not directly observed the signals are identified by their initial frequency, such as L1 and L2
OGP	[OGP]: The International Association of Oil & Gas Producers encompasses most of the world's leading publicly-traded, private and state-owned oil & gas companies, oil & gas associations and major upstream service companies. NOTE: See www.ogp.org.uk.
PDF	The probability density function of a population of data
Post processing solutions	As used herein, any processing of augmented GNSS data that takes place after the event
PPK	Post-processed kinematic
ppm	Parts per million
PPP	Precise point positioning – a global GNSS augmentation technique that corrects for GNSS satellite orbit and clock errors and employs additional modelling techniques to further correct and improve the point positioning accuracy
Precision	As used herein, defined as a measure of the random errors in observations and estimated parameters. See also Accuracy and reliability
Pseudo-range	The approximate distance between GNSS satellite and receiver derived by measuring the signal travel time. As there are accuracy errors in the time measured, the term pseudo-range is used rather than range.
QC	Quality Control
Quality measures	The measures of confidence, reliability and statistics that qualify an augmented GNSS solution. See also F-test, w-test
RAIM	Receiver autonomous integrity monitoring. An algorithmic approach that uses redundant observations to detect errors and remove them from the position solution in a GNSS receiver
Reference station	A GNSS receiver located at a precisely known location and used to determine the differential corrections employed for differential GNSS augmentation techniques. See also Tracking stations in regard to PPP solutions
Relative positioning	Satellite positioning technique that produces results relative to a reference site. See also Differential GNSS
Relative technique	Any technique using a reference station located at a precisely known location where the measurement error to each GNSS satellite can be determined
Reliability	As used herein, defined as the ability to detect outliers in observations and the impact of undetected outliers on the estimated parameters. See also Accuracy and precision
Repeatability	The variation between a receiver's observations to a particular satellite taken under the same conditions but at different times. A measurement may be said to be repeatable when this variation is smaller than some agreed limit
RINEX	Receiver Independent Exchange Format

Term	Explanation
RTCM	Radio Technical Committee (Marine)
RTK	Real Time Kinematic is a relative technique that uses the carrier phase observables for high precision positioning
SA	Selective availability – errors introduced to publicly available GPS navigation signals by the US DoD until May 2000
Satellite clock	The atomic resonance frequency standard installed in GNSS satellites. See section 3.1
Satellite orbit	The path in space followed by a satellite
SEG	The Society of Exploration Geophysicists
SBAS	Satellite Based Augmentation System. A free-to-air regional augmentation service where corrections are derived from a regional network of reference stations and transmitted to users via geostationary satellites.
S/N	Signal to noise. The ratio of the power of the received satellite signal over the background, or other, noise present
SPS	Data format designed to support land 3D seismic surveys, originally developed by Shell as the Shell Processing Support format. Adopted by SEG in 1993
Stand-alone	A form of point positioning without any augmentation. See also Absolute positioning
Static	Other than in a contextual meaning, herein means any activity where the GNSS receiver is stationary
Static post-processed	Positioning method based on the continuous and simultaneous observation of carrier phase data at two or more stationary GNSS receivers and where the solution is determined after the event by calculating the 3D baselines between one or more control points with known coordinates and the points that need to be positioned.
SV	Satellite vehicle
TEC	Total electron content. A parameter that provides a measure of the conductivity of the Ionosphere allowing models to derive transmission path delays for the satellite signals
Test statistics	The recommended test statistics for QC of GNSS position fixes are the w-test and the F-test (unit variance test). See also Quality measures
Tracking Station	A GNSS receiver located at a precisely known location and used to determine the satellite clock and orbit error for deriving corrections employed by precise point positioning augmentation techniques. See also Reference stations in regard to DGNSS solutions
Troposphere	That region of the atmosphere reaching to an altitude of approximately 40 km. See section 3.1

Term	Explanation
UKOOA	United Kingdom Offshore Oil and Gas Industry Association, trading as Oil & Gas UK, was originally known as The UK Offshore Operators' Association. It is the leading representative body for the UK offshore oil and gas industry. NOTE 1: Several of the data exchange formats referenced in this document were originally published by UKOOA. Responsibility for the maintenance of these formats passed to OGP in 2006. NOTE 2: See www.oilandgasuk.co.uk
Unit Variance	See F-Test
UPS	Uninterruptable power supply
UTC	Co-ordinated Universal Time
Vertical CRS	[ISO/TC211]: one-dimensional CRS based on a vertical datum.
Vertical datum	[ISO/TC211]: datum describing the relation of gravity-related heights or depths to the Earth.
VSAT	Very small aperture terminal, a communications satellite terminal, usually stabilised in offshore use
VBS	Virtual base station
WAAS	Wide Area Augmentation System of GPS for the USA and Canadian area
WADGNSS	Wide Area Differential GNSS, an augmentation service covering a region or wide area
WGS 84	Global geodetic reference system used for GPS
w-test	A statistical test used to detect outliers. See also Quality measures

3 Introduction and Background

Satellite-based positioning is a key positioning technology used in both land-based and offshore operations. The term global navigation satellite system (GNSS) is used to reflect the fact that multiple satellite positioning systems are available. Currently these are GPS and the Russian system GLONASS but systems from Europe (Galileo) and China (COMPASS) will become available in the future.

Standalone positions derived using individual GNSS receivers are typically not of sufficient accuracy for survey purposes. However, for E&P related surveying and positioning, augmentation data from local or global systems is used to improve data quality and to ensure a higher level of precision and accuracy than that of standalone positioning.

3.1 GNSS Signals and Error Sources

GNSS positioning techniques are based on two fundamental types of signal.

- A **code signal (observation)** is a measure of the range between satellite and receiver. The range is biased by an unknown offset between the receiver clock and GNSS system time, and hence is termed a pseudo-range. This bias is resolved as part of the position solution.
- A **carrier signal (observation)** (sometimes referred to as **phase observation**) is a measure of the number of cycles (wavelengths) of the carrier frequency between satellite and receiver. The receiver is able to measure the fraction of a cycle with high precision but does not know in which cycle the measurement takes place. If the receiver maintains lock on the carrier signal the initial estimate of this bias per satellite, termed the integer or carrier ambiguity, may be resolved.

Code and carrier signals have different properties and their use together in a positioning solution helps to overcome limitations in the individual components.

- **Height aiding** – the above techniques provide positioning using code or carrier signal observables that are generally used for horizontal positioning requirements. The accuracy associated with such methods may be reduced if the satellite geometry is poor. An approach to mitigate this effect is to use the estimated height of the antenna as an extra observable. This is known as height aiding and improves the geometry of the observables, strengthening the position fix. The introduction of height, as an extra observable into the computation, requires the user to ensure that an appropriate *a priori* error estimate is applied (perhaps based upon the expected tides) and monitors this value by checking the position quality parameters over time. During the course of a survey, changes in terrain or the tidal affects offshore may require that the height accuracy figure is adjusted.

In the future, new GNSS signals will become available from existing constellations (e.g. GPS L5). That, coupled with the introduction of new GNSS constellations, means that it will be possible to have different combinations of signals available for positioning.

The code and carrier signals are affected by a number of error sources. Some of the resulting biases can be accounted for, using either *a priori* models or estimating them from GNSS observations. Some cannot and require careful selection of antenna location in order to minimise their effect.

The approximate magnitude of the errors affecting the pseudo-range is presented in Table 2. Some of the error sources are dependent on certain factors such as the elevation of the GNSS satellites.

Sources of Range Errors	Typical magnitude (m)
Ionosphere (dependent on solar activity)	5m-20m
Troposphere (dependent on satellite elevation)	2.5m-15m
Satellite clock	1.5m
Receiver measurement noise (code only and receiver dependent)	0.5m
Satellite orbit	1.5m
Multipath (receiver dependent)	1m-5m
Geophysical effects (e.g. earth body tides, polar motion, ocean tide loading)	0.5m
Receiver measurement noise (carrier only and receiver dependent)	0.003m

Table 2 – Typical range errors in GNSS

The most important error sources are:

- **Ionosphere.** The ionosphere is the ionised uppermost part of the atmosphere ranging from 50-1000 km above the surface of the earth. The level of ionisation depends primarily on the sun and its activity. During the evening hours, the lower boundary of the ionosphere rises to 200 km above the surface as the magnetosphere turns away from the sun and fewer solar particles interact with the atmosphere. During periods of increased solar activity, ionospheric disturbances increase, causing an adverse effect on GNSS observation.

  When GNSS signals travel through the ionosphere, code observations are delayed and carrier observations are advanced, causing some of the greatest errors in GNSS observations. These errors are typically removed through application of ionospheric correction models and through use of dual frequency observations. Models use the total electron count (TEC) parameter to characterise the ionosphere allowing the models to derive delays for the signals in space. Since the ionosphere is a dispersive medium, the signal delays depend on the frequency of the signals and can be significantly reduced or even eliminated by forming a linear combination of range observations at two frequencies. It is for this reason that GPS satellites transmit signals on two carrier frequencies L1 (1575.42 MHz) and L2 (1227.60 MHz). Further details on ionospheric and other associated effects on GNSS observations are available.[1]

  The ionospheric effects affect the entire globe, but the main areas affected are the polar regions and a band extending to around 15-20° north and south of the geomagnetic equator. Work in these areas should be carefully planned to alleviate errors caused by these effects.

  Figure 1 shows the ionospheric effects for a particular GPS satellite, as computed from code observations. Also shown is satellite elevation. From this figure it will be seen that ionospheric effects depend on satellite elevation.

[1] http://gauss.gge.unb.ca/gpsworld/EarlyInnovationColumns/Innov.1991.04.pdf

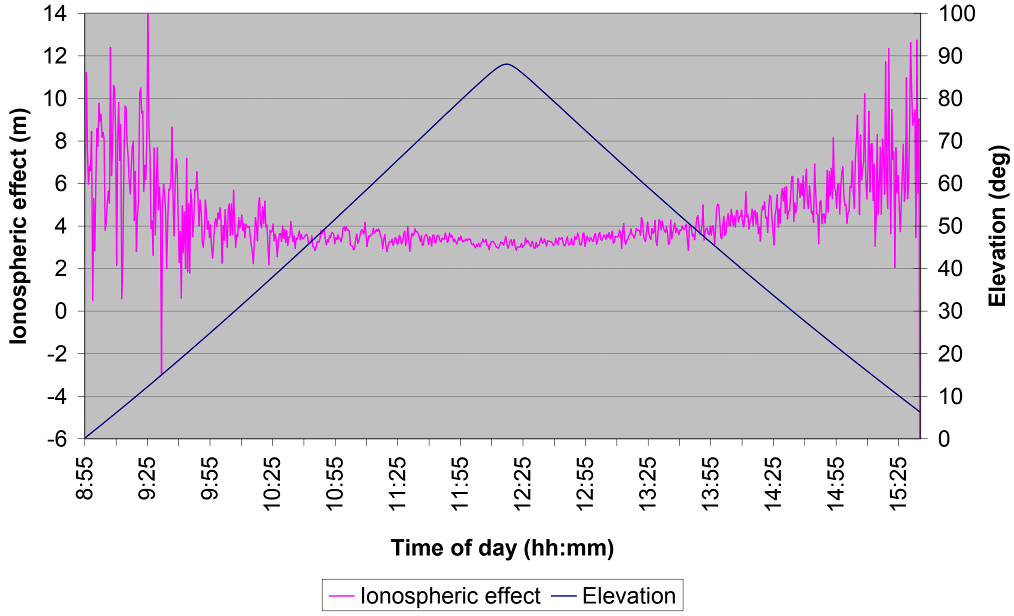

Figure 1 – Ionospheric effects, derived from code data

- **Troposphere**. The troposphere is the air immediately around the earth, from the surface to an altitude of approximately 40 km. When GNSS signals travel through the troposphere both code and carrier observations are delayed. The error is about 2.5m at zenith and roughly inversely proportional to the sine of elevation (i.e. about 15m for a satellite at 10° elevation). Delay effects are usually accounted for using a tropospheric model. Figure 2 shows estimated tropospheric zenith delays for a station at mid-latitude, as a function of time of day.

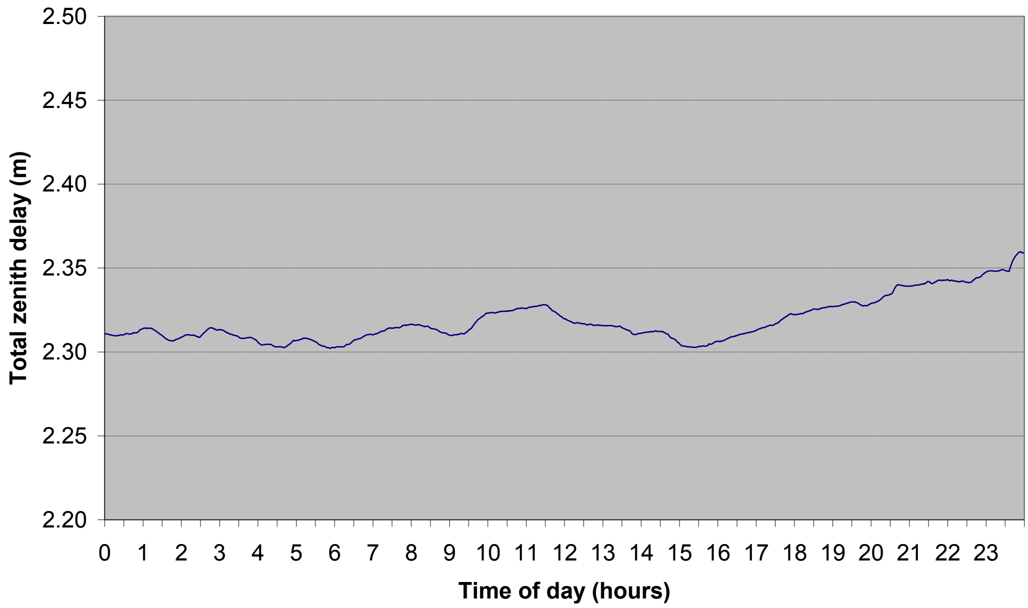

Figure 2 – Total tropospheric zenith delay, estimated at a location of 52°N, 4°E

- **Satellite clocks**: Each satellite carries its own clocks, which are synchronised to GNSS time. A satellite provides correction parameters in its navigation message, every 15 minutes, to synchronise its own time frame to GNSS time. These parameters are not perfect and are valid for only a limited period. Figure 3 shows an example of satellite clock correction error if it does not receive an update, as a function of time.

- **Satellite orbits**: Each satellite transmits a navigation message containing ephemeris parameters from which its position at any given time can be derived. These orbital parameters are not perfect; in addition, they are valid for only a limited period of time. Figure 3 gives an example of the 3D orbit error of an arbitrary GPS satellite.

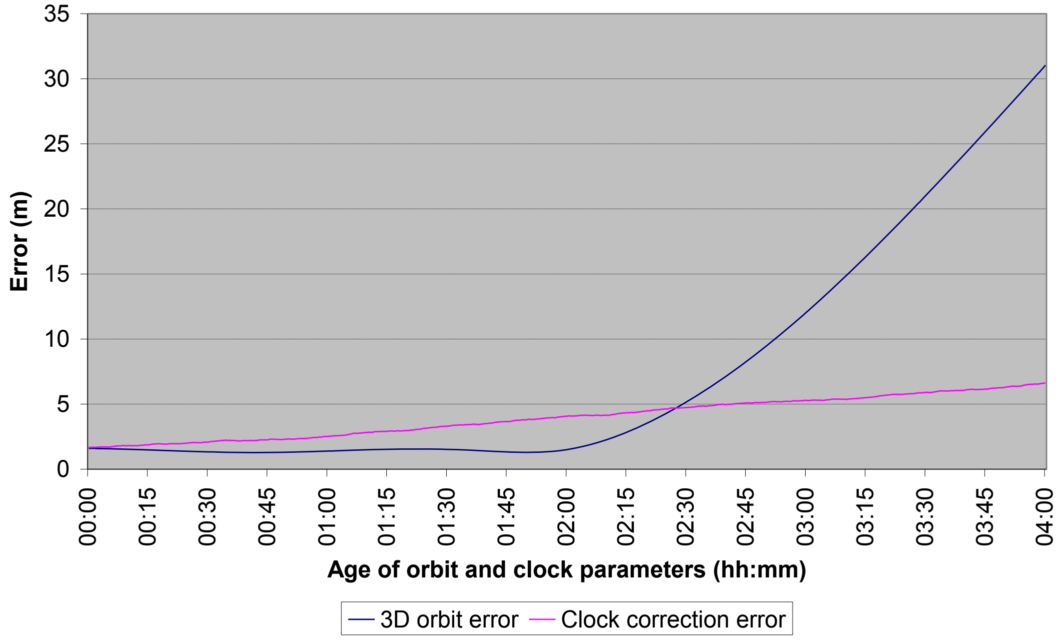

Figure 3 – Example of orbit and clock correction errors as function of age of GPS navigation message parameters

- **Multipath** refers to GNSS signals reaching the receiver antenna not directly from the satellite, but via a reflective surface such as the sea surface or a metal structure near the antenna. This mixing of reflected (indirect) and direct GNSS signals can cause errors in code observations of ~1m-5m or larger in some cases. Carrier observation errors can be several centimetres. Multipath errors can be very difficult to detect and recognise, in a static situation, observed positions will drift in a regular pattern, whereas on a moving platform such as a survey vessel multipath will look more like random noise. One indication of the presence of multipath is destructive interference between the direct and reflected path of the pseudo-range signals, giving low or variable signal to noise (S/N) ratio.

  Multipath should be avoided or reduced by careful selection of the location of the receiving antenna such that it has an open view of the sky and is clear of reflective surfaces. Antennas with ground planes and the more efficient choke rings are available; these will help to remove or cancel any reflected signals coming from below the antenna.

- **Earth tides** depend on the location of the sun and moon and mainly affect the vertical position component; the effect can be as much as 0.3m.

- **Ocean tide loading** results from the load of the ocean tides on the underlying crust; typically the effects are less than 0.01-0.02m in the vertical, but they can be in excess of 0.05m.

GNSS augmentation techniques have been developed to eliminate or reduce most of the error sources discussed above. Multipath cannot be reduced using these techniques and remains a significant potential source of GNSS biases. The various augmentation techniques are described in the following sections, and are broadly divided into relative and absolute categories.

3.2 Relative Techniques

The relative positioning technique requires a reference station at a precisely known location where the measurement error to each GNSS satellite can be assessed by comparing the calculated range against the measured range. Correction information, or raw measurement data, is then usually broadcast to the user via satellite or terrestrial radio systems.

The efficacy of the relative technique is based on the assumption that the observation errors at the user location are similar to those at the reference station. As the distance between the user and the reference station increases this assumption becomes less valid and positional accuracy decreases. This is termed spatial de-correlation. To improve robustness, data from multiple reference stations may be used.

A number of relative techniques have been developed, and these are described below.

3.2.1 Differential GNSS

There are several different implementations of the differential GNSS technique. The accuracy achieved will depend on the GNSS measurements (single or dual frequency) and observation type (code or carrier smoothed code), with DGNSS systems offering positioning accuracies ranging from 0.5m-5m at the 2 sigma level.

- A single frequency solution typically uses code observables and uses a model to compensate for the ionosphere delay. However, when the ionosphere becomes more active through increased solar activity, the model predictions are less accurate, which introduces a bias into the positioning solution. Accuracy also depends upon the observables used. Carrier observations, which are more precise (less noisy) and less susceptible to multipath are frequently used to reduce noise and bias in the code observations.
- A dual frequency solution uses observations on two frequencies to directly measure the ionosphere delay rather than relying on a model. Additional frequencies are becoming available for civilian users to offer a variety of these combinations. During normal ionospheric conditions this solution delivers accuracy similar to the single frequency solution but provides improved accuracy during periods of increased solar activity. As with the single frequency solution, use of carrier observations can provide improved accuracy.

When using multiple reference stations in a DGNSS solution, there are two broad approaches. A multi-reference or network solution uses all received correction data as (weighted) observations in the position solution. In the alternative virtual base station (VBS) approach the (approximate) location of the mobile is first used to derive one set of differential corrections, valid for that location, and these are then used in the position solution.

Use of multiple reference stations helps mitigate the degradation of accuracy due to spatial de-correlation. A guiding principle is to ensure that the survey area remains within the network of reference stations allowing interpolation of the model parameters, rather than outside where extrapolation may be required.

3.2.2 Wide Area DGNSS (WADGNSS)

The WADGNSS technique also uses a network of reference stations. However, these reference stations are not used to derive range corrections as with DGNSS but to assess three of the major sources of range error – satellite orbit error, satellite clock error and ionosphere delay. The system transmits corrections for these parameters. The orbit and clock parameters are transmitted as corrections to the parameters broadcast by the satellite. The ionosphere delay is transmitted as a correction grid.

The advantage of this technique is that spatial de-correlation is less of an issue, so the user is not so constrained by distance from a reference station. In addition, a single frequency user does not have to rely on an ionosphere model.

The accuracy of the solution is constant when operating inside the network of reference stations but degrades towards the outside.

3.2.3 Real Time Kinematic (RTK)

RTK involves radio transmission of L1 and L2 carrier phase observation data or corrections from a base station of known co-ordinates, to a mobile receiver where a position is determined. Double-differencing the measurements and resolving the integer ambiguities provide relative positioning accuracies at the centimetre level. The carrier phase observable of RTK supports the calculation of accurate three-dimensional positions.

The range over which RTK can operate is limited, often to 20km or less, due to error decorrelation affecting the derivation of the carrier phase integer ambiguities and radio transmission limitations. The range can be extended by not resolving the integer ambiguities but this will decrease the accuracy of the position solution. This is sometimes referred to as the Float solution.

An alternative to the single station RTK is to use **network RTK**. This technique utilises a network of permanent GNSS receivers where data from all stations is combined and used to generate RTK corrections or raw data for a mobile user. The mobile receiver connects to the network RTK server via a one-way or two-way communication link (e.g. cell phone) in order to receive the RTK correction data. Once the receiver has this data it will compute a position of centimetre accuracy dependent on the number and distribution of stations. The networks vary in size from small local networks consisting of a few reference stations to large country-wide networks. Commercial network RTK systems are operating in various countries all over the world, typically in populated areas of developed countries.

3.2.4 Post-Processed Positioning

Static positioning is based on the continuous and simultaneous observation of carrier phase data (and sometimes code data as well) at two or more stationary GNSS receivers to derive the 3D baseline between these. At least one receiver will be placed at a point whose co-ordinates are known, whilst the others will be placed at points whose co-ordinates are required. During post-processing, these position parameters, together with carrier ambiguities, and often other parameters as well, such as tropospheric effects, are estimated. Observation data is usually logged at 15 or 30 second epochs. The required observation period for static GNSS depends on the baseline length. For short baselines it is usually easy to quickly resolve the carrier ambiguities to their integer values and the observation period can be short to estimate positions with high accuracy. For longer baselines, it is often not possible to resolve the integer ambiguities, in which case it is necessary to rely on the change in receiver-satellite geometry to separate position parameters from ambiguities and obtain the desired accuracy. In practice, this means an observation period of at least one hour. Typical accuracies for static baselines are of the order of 3-10mm (1 sigma) in each component (X, Y, Z) plus 1-2 ppm of baseline length.

Post-processed kinematic (PPK) positioning also relies on the collection of continuous and simultaneous (code and) carrier phase data to determine the location of a moving receiver. As with the static solution, at least one receiver will be placed on a location of known co-ordinates. Observation data is logged at a much higher rate than for the static case, usually once a second. However, for long baselines, for which ambiguities cannot be resolved, it is still the change in geometry that determines the final accuracy. To overcome the initial convergence period, smoothing (reverse processing) of the data may be required.

Many countries now operate a network of permanent reference stations which continuously log GNSS data and this data is usually made available for users to download from internet sites.

3.3 Absolute Techniques

In its most basic form, where augmentation has not been used to improve the positioning solution, this method is commonly referred to as 'stand-alone' GNSS as it relies only upon observations between the GNSS satellites and the receiver station.

Where absolute techniques incorporate GNSS augmentation, in contrast to the relative positioning technique, the solution does not require local reference stations to determine differential corrections. Instead, techniques are employed that use the data from a network of GNSS reference (tracking) stations to model and correct for errors in the satellite navigation data.

3.3.1 Precise Point Positioning (PPP) Technique

Real time Precise Point Positioning (PPP) is an absolute method which uses augmentation data in the form of satellite clock and orbit corrections. These corrections improve the accuracy of the standard navigation messages broadcast by the satellite. The augmentation data is derived from a global network of GNSS tracking stations. A single set of globally valid orbit and clock corrections is generated for the entire GNSS constellation and broadcast to the PPP user community. For this reason the technique is not subject to spatial de-correlation.

A dual-frequency receiver is required to facilitate calculation of the local ionosphere delay, leading to a high accuracy solution. This means that the system can be employed in all areas including those experiencing high ionospheric activity. The effects of multipath and GNSS receiver noise are reduced through use of the dual-frequency carrier phase observables. Tropospheric error is estimated using a tropospheric model and any residual effect is modelled as part of the solution. The PPP technique can provide a horizontal positioning accuracy typically better than 15cm (2 sigma 95%) and a vertical accuracy in the order of 15-25cm (2 sigma 95%).

PPP solutions require a short period of initialisation. From a cold start, depending on the GNSS satellites in view, convergence to full PPP accuracy is typically less than 20 minutes with ~25cm (2 sigma) accuracy achieved within 10 minutes.

3.4 Vertical Component

In the early days of GNSS, precise positioning was limited to the horizontal components (x and y ordinates). The steady improvement in augmentation techniques and receiver capabilities, particularly, means the height component (z ordinate) has become more accurate and a common requirement.

The absolute vertical (height) component of a GNSS antenna is in terms of the GNSS reference ellipsoid, e.g. WGS 84 in the case of GPS, i.e. the height (h) of the antenna above the ellipsoid. To derive the orthometric height (H), that is, the height of the antenna above the geoid, requires knowledge of the separation between the geoid and ellipsoid (N) at the place of observation. The formula is:

H = h - N

Careful attention to the sign of the values is critical. Figure 4 below illustrates the vertical elements.

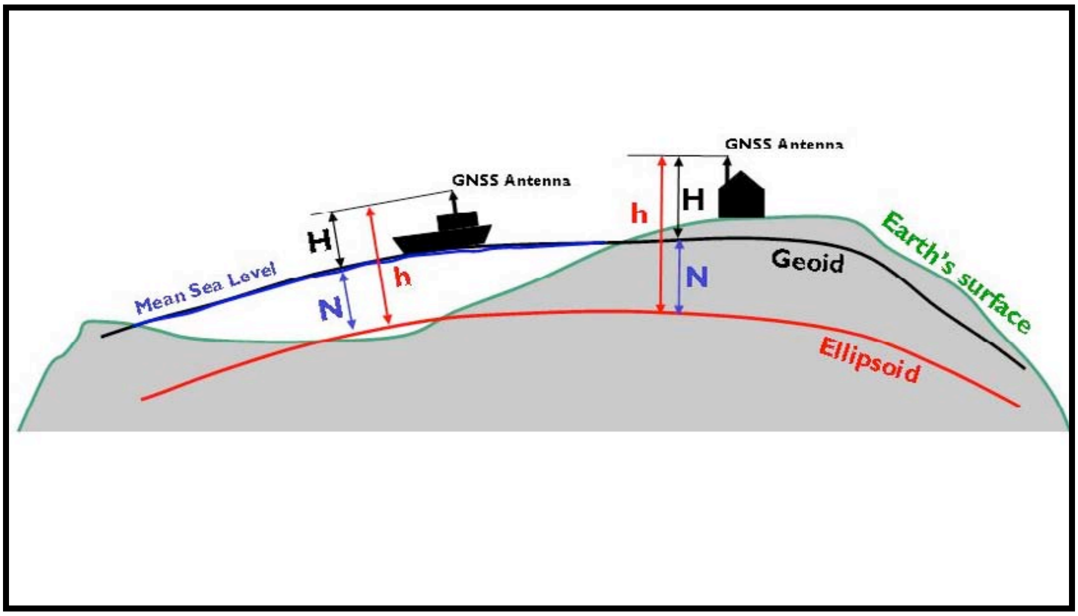

Figure 4 – Absolute vertical height of an antenna

A regular grid of geoid-ellipsoid separation values (N) is described as a geoid model. A number of models exist, and choice of an appropriate model will be a consideration when using GNSS to derive orthometric heights. This subject is discussed in more detail in section 4.2.

3.5 GNSS Augmentation Service Providers

Several sources of augmentation data currently available are described below.

3.5.1 Commercial Suppliers

Commercial service providers offer access to augmentation data broadcast via leased communication satellite channels or through terrestrial radio broadcast. A key aspect of the service is redundancy, which is an important consideration for E&P operations. The commercial service provider continuously monitors the quality of satellite signals and correction messages and also provides access to a 24-hour support network. Service providers typically offer products delivering a range of accuracies from decimetre to metre level, and several operate on a global basis.

3.5.2 Free-to-Air Satellite Based Augmentation Systems (SBAS)

There are several satellite based systems delivering free-to-air augmentation data to GNSS users on a regional basis. The concept uses corrections derived using a regional network of reference stations.

Systems provide accuracy typically better than 3 metres (2 sigma) but users are cautioned that the accuracy achieved is variable and is at the user's risk. Augmentation systems implemented, or soon to be implemented include:

- WAAS (North America) – Wide Area Augmentation System operated by the United States Federal Aviation Administration;
- EGNOS (Europe) – European Geostationary Navigation Overlay Service operated by the European Space Agency;
- MSAS (Japan) – Multi-functional Satellite Augmentation System operated by Japan's Ministry of Land, Infrastructure and Transport;
- GAGAN (India) – a system under development by India.

3.5.3 Marine Radio Beacon (IALA)

For the purposes of coastal navigation, free-to-air transmission of GNSS correction signals are broadcast over local marine medium frequency (MF) radio beacons. This service is under the auspices of the International Association of Marine Aids to Navigation and Lighthouse Authorities (IALA). The local radio beacon networks are operated by the lighthouse authority, port authority or coastguard of any given country. Corrections are broadcast in a band allocated for maritime radio navigation. The coverage varies depending on the transmission power and is typically 200-350km but is subject to short and long term variation due to environmental and seasonal conditions and is unmonitored. IALA is guaranteed only to meet statutory navigation accuracy of 8 metres, although in reality, with Selective Availability (SA) no longer implemented, the accuracy is typically better than 5 metres at the 2 sigma level.

3.5.4 Local Stations

For projects that cannot utilise available services an option would be to install a local reference station to provide augmentation data for the work location. Such reference stations would be equipped with a GNSS receiver, configured to transmit either DGNSS corrections or RTK observations via a radio link or cell phone to the mobile receiver.

Information on installation, co-ordination and control of the station, which is critical to ensure high accuracy data, is available in section 4.

3.6 Summary of Systems Available to Users

The following table enables the reader to compare the various GNSS techniques and their approximate accuracies and availability. However, this should be treated with care, as anything but the most basic comparisons are susceptible to exceptions and variation in receiver hardware, algorithms and approaches used in positioning solutions, and the modernisation of GNSS. This document does not provide any detailed GNSS technique/system comparison, as the associated technology evolves so rapidly that the document would require almost continuous revision. Similarly, there are no recommendations for the use of particular GNSS techniques or services to satisfy specific positioning requirements.

	Relative technique						Absolute technique					
System	MRB (IALA)	SBAS	Single station DGNSS	Network DGNSS	Single station RTK	Network RTK	Static post-processed	Dynamic post-processed	Stand-alone	PPP	Static PPP (post-processed)	Dynamic PPP (post-processed)
Coverage	Local	Regional	<2000km	Regional	Local	Local	Regional	Local	Global	Global	Global	Global
Real time delivery system	Radio link	Satellite internet	Satellite, radio link, cell phone	Satellite, radio link, cell phone	Radio link, cell phone	Radio link, cell phone	n/a	n/a	GNSS satellites	Satellite internet	n/a	n/a
Typical horizontal accuracy (2σ)	~3m	~3m	~1m	~0.5m	2cm +1ppm	5cm +1ppm	5mm +1ppm	2cm +1ppm	5-10m	~15cm	0.5 -1cm	5cm
Typical vertical accuracy (2σ)	~5m	~5m	~2m	~1m	5cm +1ppm	10cm +1ppm	10mm +1ppm	5cm +1ppm	10-15m	~20cm	1-2cm	10cm
Spatial de-correlation	Subject to spatial de-correlation of errors and affected by number and density of reference stations						Not subject to spatial de-correlation of errors					
	Availability (dependent on installation, coverage and delivery system)											
Land	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Inshore (4km offshore)	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Coastal 4-20km	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Offshore 20-350km	✓	✓	✓	✓	✓/×	×	✓	✓	✓	✓	✓	✓
Offshore 350-2000km	×	✓	✓	✓	✓/×	×	✓	✓	✓	✓	✓	✓
Oceanic	×	✓/×	×	✓	✓/×	×	✓	×	✓	✓	✓	✓
>76°N/S latitude	×	×	✓	✓	✓	×	✓	×	✓	✓	✓	✓

Table 3 – Summary of positioning services

4 Installation and Operation

The following section provides information and guidance in the selection, installation and operation of appropriate satellite positioning systems for both onshore and offshore positioning.

4.1 Considerations for the use of Satellite Positioning

4.1.1 Preparatory Work and System Specification

The following factors should be considered when specifying or selecting an appropriate GNSS system to meet the requirements of the user or project: the required accuracy, work area, level of redundancy, vehicle/vessel activity (number, speed, operation) and other considerations such as: the atmospheric conditions in the general area of work; the sources of signal interference or masking and finally the application of the positioning data. In some circumstances the user may not be able to select the satellite positioning system of their choice and must select from those available.

The type and nature of the installation of the mobile equipment should also be considered. Whether the installation will be permanent or short-term will influence the placement of cables and the location of antennas. Planned maintenance should be taken into consideration for permanent installations.

4.1.2 Required Accuracy

Accuracy of the satellite positioning system can be the principal driver in the selection of an appropriate system. It is important that the entire error budget for a survey is considered so as to generate a realistic assessment of the overall performance. This includes the positioning accuracy, antenna offsets and their dimensional control and for offshore surveys, subsea positioning accuracy. This last element is becoming recognised as more critical as the industry moves into deeper water.

It is also important to consider whether the accuracy required is for the horizontal and/or the vertical component as this can influence the choice of positioning system. Examples include derivation of water depths during offshore surveys and dredging and for profiling routes and clearances on land.

Once the required accuracy has been established for the surface positioning component, the most appropriate positioning solution can then be selected. See Table 3 in section 3.6.

Section 5 outlines how accuracy can be determined and monitored to ensure that the satellite positioning system meets the required specification.

4.1.3 Geographic Operating Region

A critical consideration is the geographic location of the work area. As a consequence of the orbital configuration of GNSS satellites, coverage at high latitudes is not as optimal as that experienced at the equator, as the satellites never rise above 64° elevation relative to the user. There may be sufficient useable satellites but they will all be at a lower elevation when compared to the coverage at the equator. This will cause periods of increased dilution of position (DOP) due to the poorer satellite to receiver geometry and signals will be more susceptible to the effects of the atmosphere and multipath. Mission planning should be undertaken to ensure suitable satellite coverage, accuracy and reliability are available.

At high latitudes choice of GNSS augmentation systems may be limited. Furthermore, accuracy may be degraded by the increased distances to, for example, DGNSS reference stations. Consideration of reference station coverage should be included at the planning stage.

The geostationary communication satellites used to deliver GNSS correction data can typically be used up to latitudes of 75-78° north or south. In work areas above the latitude horizon of the communication satellites they may no longer be used for correction data delivery. In these instances alternative means of delivering correction data will be required.

Working in polar regions presents difficulties due to geomagnetic disturbances which are intensified during periods of increased solar activity. Work areas near the geomagnetic equator could, in periods of increased solar activity, also be affected by increased ionospheric disturbance. See section 3.1.

4.1.4 Level of Redundancy

It is recommended that at least two fully independent positioning systems are installed and available. Although the concept of independent positioning is sound it is difficult to realise in practice as there is often a common element in terms of the satellites themselves as most commercial systems are mainly based on the GPS system. However solutions that are based on GPS or GLONASS can be considered fully independent if the associated augmentation systems are fully independent. In addition within each GNSS system and the associated augmentation, there are multiple redundancies available. In theory multiple satellites could fail and accurate positioning would still be available. This redundancy can be further increased through the combined use of different GNSS systems. In due course the option of GPS, GLONASS, Galileo and COMPASS systems will bring even greater availability, reliability and independence.

When considering redundancy of systems installed, the following should be considered:

- Redundancy of hardware (including spares): Typical installations will require that a minimum of two independent satellite positioning systems are installed. Additional spare systems could be installed (spare systems should be installed with 100% spare components) and kept offline until needed.
- Redundancy of augmentation delivery links: Most installations can provide redundancy in the data link used. Separate and individual data links may offer some protection from interference or atmospheric conditions. Often the selection of a different data link introduces the option of also selecting a different positioning technique.
- Redundancy of positioning methods: Two or more independent systems provide the potential for different algorithms and techniques to be used.

It is not recommended to share satellite positioning systems between survey applications and vehicle/vessel navigation system, as the configuration requirements of the satellite positioning may be different for each use. The use of a shared system also raises concerns about the susceptibility of the vehicle/vessel navigation in case of a failure of the satellite positioning system. Positioning systems do not have the same regulatory and safety critical requirements as navigation systems. If there is no alternative, particular care should be exercised to ensure the system meets the specifications of both parties without affecting the operational requirements of either.[2]

Onshore static stations often require a similar level of redundancy to that stated above, as they are often critical to 24-hour operations as base-stations. The redundancy requirements for mobile units may be less stringent, due to the ease of access to spare equipment and services, and the effects of equipment failure but it is recommended that 100% spares are carried for at least one full GNSS system.

4.1.5 Level of Support

The available level of service and technical support should be considered when selecting a positioning system. Free-to-air services generally offer no support whereas commercial service providers may offer a dedicated support network, service engineers and system performance monitoring.

[2] See for example IMCA S 009 – Guidelines for the shared use of DGPS for DP and survey operations

4.2 Geodetic Considerations

These guidelines do not provide an in-depth appreciation of geodesy, this section provides guidance on geodetic reference system issues that should be considered when planning GNSS positioning[3].

The coordinates output by augmented GNSS receivers using the reference station networks of the commercial wide area differential GNSS typically refer to the International Terrestrial Reference System (ITRS). The ITRS is realised through a global network of sites for which coordinates and velocities at a specific time are recalculated/readjusted on a regular basis to account for tectonic movement and other changes such as earth orientation in celestial space. Each coordinate set is referred to as a reference frame for the beginning of a calendar year, for example ITRF 2005 and ITRF 2008. These ITRF solutions include station velocities to allow station coordinates to be calculated at time of survey, the so-called Epoch Date. It is then possible to refer coordinates to one of the frames at a particular date. For example, a survey conducted on 1st June 2010 (42% through the year) might refer to ITRF2008@2010.42.

The GPS system has its own CRS, WGS 84. WGS 84 has been refined several times; the version currently implemented by the GPS Master Control Station and in the National Geospatial Intelligence Agency of the USA (NGA) orbit processing is consistent with the ITRS at the decimetre level.

The coordinates output from the Russian GLONASS satellite navigation system are referenced to a global reference system Parametri Zemli 1990 (PZ-90) which is slightly different to WGS 84. It too has been refined such that the latest realisation is comparable with ITRS at the decimetre level. This difference is reconciled either in the dual GPS/GLONASS satellite receivers or in software in the dedicated positioning computers used for survey operations, and coordinates output in a unified WGS 84. Users should check that the methods adopted by such systems do in fact address the issue of the small differences between the coordinate reference systems.

The differences between PZ-90 and ITRS are much smaller than the current precision of the orbit parameters transmitted by the GLONASS satellites. For real time differential GNSS positioning applications they will typically be reduced to the level of a few centimetres for distances up to 1000 km from a single reference receiver. Consequently some techniques may not apply any coordinate transformation between PZ-90 and WGS 84. For PPP applications, the precise positions of both the GPS and GLONASS satellites are resolved in the solution, so in such applications no transformation is required.

The CRS for coordinates output from the European Galileo satellite navigation system is the Galileo Terrestrial Reference System (GTRS), which is aligned to within a few centimetres of the ITRS.

E&P related surveying and positioning operations are typically referenced to the local national or regional geodetic coordinate reference system (CRS). In Europe, North America and other areas where the historic local geodetic CRSs have been replaced with modern geodetic reference systems tied to the ITRS, the oil industry may still use the legacy local system. The primary reason is to ensure backward compatibility with historically acquired survey data. In such circumstances the coordinates output from the GNSS need to be transformed to the local CRS. These transformations are often colloquially referred to as 'datum shifts', but this terminology is not correct.

A number of different coordinate transformation methods exist. Refer to the EPSG Geodetic Parameter Dataset and Guidance Note 7 part 2 (OGP document reference 373-7-2) for a description of these various methods. Particular care is needed with 7-parameter geocentric transformations where two conflicting conventions for the sign applied to axes rotations are in widespread use. These are the "Position Vector transformation" and the "Coordinate Frame Rotation" methods. Either may be referred to as "Bursa-Wolf". It is imperative to know which of the two rotation conventions the GNSS software uses, and to which method a given coordinate transformation refers. Additionally it is important to know the accuracy and the area of use of a given coordinate transformation.

Historically the oil industry has had access to a number of sources of coordinate transformation parameters for an area, region or country. Most of these will be available from the EPSG Geodetic Parameter Dataset,

[3] OGP Geomatics Committee Guidance Notes:
Note 4 – use of the International Terrestrial Reference Frame (I.T.R.F.) as a Reference Geodetic System for Surveying and Real Time Positioning. http://www.epsg.org/guides/g4.html
Note 7 – EPSG Geodetic Parameter Dataset supporting information Parts 1 through 4. http://www.epsg.org/guides
Note 13 – Advisory note on derivation of geodetic datum transformations and use. http://www.epsg.org/guides/g13.html

the de facto industry standard dataset of parameters for coordinate reference systems and coordinate transformation maintained and published by the OGP Geodesy Subcommittee. Coordinate transformation parameters will have a limited spatial validity and may be of varying quality depending on age, origin etc. The EPSG Geodetic Parameter Dataset includes information concerning area of use and accuracy.

When first planning survey and positioning operations in a new area it is recommended that reference is made to the EPSG Geodetic Parameter dataset to determine if a coordinate transformation of sufficient accuracy has already been established for the area. Some, typically older, transformation parameters may not have sufficient accuracy. It may therefore be necessary to plan a new observation campaign to derive more accurate transformation parameters. Although the absolute accuracy of existing transformation parameters may not be the best possible, the high relative accuracy achievable with augmented GNSS is still retained. Although the absolute accuracy may be improved by applying new and more accurate transformation parameters it is often necessary, or preferred, to use the same transformation parameters as applied historically to ensure consistency with, and high accuracy relative to, existing survey results from the area.

If new transformation parameters are established for an area it is recommended that these are published to the oil and survey industry at large through the EPSG Geodetic Parameter data set to avoid multiple geodetic control surveys by companies operating in e.g. neighbouring blocks or areas. If there is a suitable transformation promulgated by the national mapping authority, this should be used in preference to developing a new transformation. For example in Canada, the NTv2 transformation is recommended unless compatibility with a previously adopted alternative is required.

4.2.1 Vertical Datum

The vertical component output by a GNSS receiver refers to the reference ellipsoid (see section 3.4). To derive the gravity-related height above the vertical datum used for survey and positioning, a transformation using a height correction or geoid model is required. These models may be global or local and they may have been refined over time to improve the vertical accuracy.

When planning survey and positioning operations the vertical datum/geoid model should be considered with respect to vertical accuracy requirements and also if consistency with, and high accuracy relative to, existing survey results is required.

4.3 System Components and Description

For a satellite positioning system installed in a mobile vehicle/vessel, the components can be split into two distinct parts: the equipment installed externally and the equipment installed internally.

The external equipment typically consists of the GNSS antenna, HF/MF and satellite communication antennas, RF (coaxial) cables and connectors. It is important that the antennas are installed correctly and that all connections are watertight to prevent degradation of the system.

The internal equipment typically consists of the GNSS receivers and augmentation data demodulators, data cabling, data connectors, positioning computers, computer systems running QC software applications, and often an uninterruptable power supply (UPS). The equipment and level of hardware redundancy varies across all types of survey, and is largely influenced by project requirements (the risk and potential impact of equipment failures are important considerations). It is recommended to operate at least two fully independent GNSS installations backed up with 100% spares (see section 4.1.1 for further information), to mitigate the impact of system failures.

The components used in an RTK system differ as augmentation data demodulators and satellite communications are not required and handheld devices are typically used as the QC and positioning computer. The radio-based data link between base stations and all mobile RTK devices will often introduce additional equipment requirements, with repeater stations commonly used if the work area is large and mountainous and distances between the rover and base stations increase beyond VHF range. It is advisable to have spare equipment available for at least one full GNSS system (with spare components to enable use either as the base station or the rover), but common throughout many terrestrial surveys, this is not usually installed until it is required.

Normally the equipment used in onshore applications will have less spares than in the offshore environment, due to easier access to replacements and services and the generally lower impact of equipment failures on a project. Large projects may not have 100% redundancy but will be required to have sufficient spares in case of multiple failures. Some level of hardware redundancy will be required and is usually determined by the location of the work area (extreme conditions, remoteness) and the type of work being undertaken (single or multiple units). There are exceptions to this, however, where full redundant GNSS systems are commonly operated at (semi) permanent base stations.

4.4 System Installations

Once the appropriate satellite positioning system has been selected, one of the most important issues is the correct installation of the system to ensure that it functions as expected.[4] Incorrect or inadequate installation can lead to poor positioning performance or complete loss of position. The following sections address the three phases to the installation and commissioning of a satellite positioning system on a mobile platform (vehicle/vessel).

While the following is specific to a mobile installation, the same process can equally be applied to the installation of a local RTK/DGNSS reference station.

4.4.1 Pre-Installation Phase

The pre-installation phase covers all the preparatory work and planning undertaken prior to the installation. This should be fully agreed by all parties and be approved by the vehicle/vessel owners and operators.

The following should be addressed during the pre-installation phase:

- antenna location should, as far as possible, be free of any potential obstructions;
- identification of potential sources of interference;
- arrangements for any pre-fabricated components such as antenna frames or mast structures;
- types and lengths of cable required for use with each antenna;
- connection types and any ancillary components;
- cable routes;
- offset measurements of the antennas to the common reference point (CRP) or other points of interest on the vehicle or vessel;
- location of internal (below deck or in-vehicle) hardware components and any prefabricated mountings;
- power distribution to hardware units and power cabling requirements;
- redundancy plan for power distribution;
- interfacing details including data telegram formats, protocols and cabling requirements;
- system configuration including:
  - services in use
  - delivery links
  - reference station selection
  - operating parameters such as maximum age of correction data, elevation mask etc.
  - verification checks.

As part of the pre-installation phase it is useful for a mobilisation procedure to be produced as this provides the installation engineer with the necessary information and guidance for completing the correct installation onboard the vehicle/vessel. It also allows all relevant parties to understand and agree the scope of work prior to the installation phase.

[4] IMCA S 012/M 199 – *Guidelines on installation and maintenance of DGNSS-based positioning systems*

4.4.2 Installation Phase

Typically when installing a satellite positioning system the following steps are conducted:

- antenna installation (see details below);
- cabling and connections between antenna and the internal equipment;
- fitting of hardware and software;
- interfacing of hardware and software to the integrated survey computer system and peripherals;
- configuration of hardware and software;
- performing post-installation checks involving (in the offshore environment) the use of transmitting devices to ensure that nothing is interfering with the satellite positioning system.

A mobilisation report should be provided as a record of how the equipment was installed and configured to help with maintenance and support.

Antenna Installation

The location of the antenna is the single most important aspect of the successful installation of a satellite positioning system and there should be no compromise on quality. Much relies on the antenna installation; if this is not correct or is of inadequate quality then it is unlikely that positioning requirements will be met.

If the antenna is installed in a poor location it can suffer from masking, multipath or interference from other radio sources which can affect the positioning performance.

Figure 5 – Examples of a good (left) and bad (right) antenna installation

On vessels and vehicles, the number of systems requiring antennas has increased but unfortunately the available mast or mounting space has not. Therefore, there is competition for the best locations for these antennas, generally the highest point, such as on a vessel's mast. Nevertheless, since the survey operation is reliant upon accurate positioning, installation of the DGNSS system antenna should, as far as is practical, take precedence. Lightning strikes could become an issue in certain parts of the world and it may be prudent to consider some form of lightning protection device.

On vessels or vehicles where positioning is recognised as critical to the operation, dedicated or separate positioning mounts may be fitted, primarily for DGNSS and associated antennas.

4.4.3 Commissioning/Operational Phase

Once the satellite positioning system has been installed and configured the following should be checked:

- the satellite positioning system is operating correctly and to specification;
- offsets between GNSS antenna and survey equipment and sensors are measured accurately and applied correctly;
- co-ordinate transformation parameters are applied correctly.

Offsets and co-ordinate coordinate transformation parameters are typically entered as survey parameters into an integrated positioning system used to compute and record sensor positions online and in real time.

Once operational and commissioned, ongoing maintenance of the satellite positioning system should be undertaken to ensure correct operation throughout its lifetime.

Antenna Offsets

All GNSS measure the position of the receiving antenna.

During E&P related surveying and positioning activities, GNSS is typically used to determine the positions and heights of the survey equipment and sensors used to record the survey data (e.g. seismic source and receiver locations during a seismic survey, echo sounder transducer on a bathymetric survey), and to determine the positions of points of interest (e.g. the drill centre position during a rig move). Except for static surveys these positions are typically computed online and in real time by an integrated positioning system.

As the location of the GNSS antenna rarely coincides with the location of the survey equipment and sensors and points of interest being positioned, the offsets and headings between antennas and sensors/points of interest must be known; on vessels, the heading between the GNSS antenna and points of interest is typically observed by gyrocompass or GNSS based heading sensor.

The offsets must be measured accurately in all three dimensions and are typically measured by conventional land survey methods from a common reference point (CRP), e.g. the centre of gravity of a survey vessel, the centre of the drill floor on a drilling rig. The offsets must be measured again when a GNSS antenna is moved or when a new survey system is installed. The offsets should be documented and that documentation should be readily available for future reference; offsets should be verified annually and also before starting a new job.

Coordinate Transformation

Where the survey's coordinate reference system (CRS) differs from the GNSS's CRS (see section 4.2), the computed position needs to be transformed from GNSS CRS to survey CRS. Typically the transformation is conducted online and in real time by the integrated positioning system or offline during post processing of the data. The coordinate transformation parameters and the transformation computation should be verified before starting a new job.

Alongside Verification of Installation and Survey Parameters on Dynamic Vessels

Alongside verification should be conducted to confirm correct system operation, antenna offsets and co-ordinate transformation:

- on completion of GNSS installation;
- where any physical aspect of the GNSS installation has been altered (or where it cannot be demonstrated that there have been no alterations);
- prior to starting a new job.

Antenna offsets should be checked by repeat measurements against the installation survey documentation and general arrangement drawings.

System operation and co-ordinate transformation should be checked by recording the following data, with the antenna static (as far as is practicably possible), over a period of at least 30 minutes:

- the individual positions output from all the satellite positioning systems (on vessels there should be at least two independent systems);
- the calculated positions of the antennas, CRP and offset points for each individual satellite positioning system, transformed to local/survey CRS;
- the position of GNSS antenna and representative offset points observed by conventional land survey methods from known control points in the local/survey CRS;

The positions of antennas, CRP and offsets in the local datum are calculated for each GNSS by the integrated positioning system. These positions are then compared with the corresponding positions determined from a known control. This is done to verify that the satellite positioning systems are operating properly and are not affected by multipath and other errors, and that co-ordinate transformation from satellite to survey CRS is conducted correctly. This process will also inherently verify the measured offsets between the antenna and points that are compared.

Where the survey CRS is the same as the GNSS CRS and verification of co-ordinate transformation is not required, GNSS antenna position can be verified by post-processing of raw GNSS data logged in RINEX format. This cannot be used to verify the offsets of GNSS antennas, survey equipment and other points of interest on the vessel.

In-Field Verification of Installation and Survey Parameters on Dynamic Vessels

Where practical, alongside verification should be supplemented by in-field verification, to confirm correct and appropriate positioning system functionality. For projects where the vessel is mobilised offshore in-field verification should be conducted.

There are various methods available for in-field verification of GNSS installations, by direct comparison of the GNSS derived positions with objects/infrastructure at known locations; either infrastructure above the sea surface - for example, by transiting a drilling rig or a platform; or more commonly by determining the position of subsea infrastructure, for which the vessel requires appropriate subsea positioning equipment (seabed installations with known location can be surveyed by echo sounder or side scan sonar). The position of the known structure can then be calculated from the observed GNSS and survey data and compared with the known as-built position. Any significant discrepancies will indicate possible offset or co-ordinate transformation errors.

The methods are typically only accurate at the 5-10m level and are only suitable for checking the positioning system for the presence of gross errors. The verification process alone will not determine the source of the error, which will require further investigative work.

In-field verification should also include aspects of continuous monitoring of the GNSS installation(s), by continual comparison in real time of the positions determined by the different satellite positioning systems available (a minimum of at least two fully independent satellite positioning systems should be operated in parallel). Comparison between the independently derived vessel positions gives a good indication of real time system performance and can help to identify periods of deteriorating positioning accuracy. All subsequent survey sensors positioned relative to the GNSS can be monitored in a similar manner, and this can be useful in the assessment of positioning accuracy and performance of the full survey spread.

PPP GNSS solutions can periodically monitor the system performance by recording raw GNSS RINEX data and post-processing this data to produce a PPP solution that can be directly compared to (and thereby verify) the recorded real time-derived DGNSS vessel position, albeit not in real time.

If the verification process (either alongside, in-field or continuous monitoring method) identifies the presence of gross errors that degrade the GNSS positioning accuracy beyond project requirements, survey acquisition should be stopped. Once the source of the error has been identified and the error removed, the verification process should be repeated, which may require additional verification measurements to alternative reference points and repeat checking of all software configurations, hardware installation and offset measurements. It is not recommended that any of these verification options be used alone, but that they are used to complement each other and form a process of effective and continuous monitoring of the GNSS system performance and integrity, such that any installation or configuration changes are appropriately managed.

Static/Land Survey Verification

For GNSS equipment used for static/land survey work certificates of calibration or current service records should be provided for all principal surveying equipment before the job commences.

The verification of mobile survey equipment by comparison of the GNSS derived position against known control points should be performed twice daily (at the start of daily survey operations and at the end of daily survey operations).

4.4.4 Ongoing Maintenance

For permanently installed systems planned preventative maintenance should be ongoing. Irrespective of how good the installation is, it will deteriorate over time, particularly those parts of the installation exposed to the elements.

It is recommended that an installation undergoes a complete re-evaluation at least once per year, with any subsequent remedial work completed as soon as practical thereafter.

The key components that should be checked are:
- antenna – for signs of damage;
- antenna connections;
- antenna offsets;
- antenna cable integrity including any change in signal to noise values;
- interfacing protocols and cabling;
- condition of all components;
- upgrade all software and firmware to latest versions;
- all configurations.

4.4.5 Recording Changes to Installations

It is important that any changes made to an installation or its set-up configurations, including software changes, are formally reported and recorded. This is particularly important where shift changes are used and also at crew changes, so that everyone is fully aware of what changes have been made and why.

A formal change document should be maintained and available to authorised personnel detailing:
- installation procedures, including any difficulties encountered;
- location drawings of all the system elements, including cable runs (updated at every change);
- system configuration(s) together with diagrams and offsets (updated at every change);
- details of the changes explaining where and why the changes were made;
- at every shift or crew change, a 'toolbox talk' should be completed by the handover crew. Where a change management system is in place, the change document should be included for reference.

4.5 Operational Awareness

In the operational environment it is important that the user is aware of influences on the accuracy and availability of satellite positioning, including configuration changes or changes to the operational environment. These factors can lead to performance degradations, signal interruptions or loss of service. Occurrences can be infrequent and impact only temporarily on the quality of the positioning, making their detection and isolation difficult in real time.

The following table presents some examples of causes that can impact the performance of positioning systems and offers some means of mitigation.

Effect	Cause	Mitigation
Interruption to SV tracking	Multipath or obstructions to antenna such as when working close to platforms or other structures	During installation ensure antennas have clear line of sight to the sky At mission planning stage check whether obstructions will impact performance Consider use of multiple GNSS systems to provide additional satellites (e.g. GPS and GLONASS)
Radio-frequency interference	Transmitting devices such as data or video telemetry systems or satellite communication systems (e.g. VSAT, Sat-C) interfering with GNSS and satellite correction delivery system	Antennas should be installed at the maximum distance from other radiating antennas (IMO recommendation 3m separation from other radiating sources)[6] Ensure antenna cables are terminated correctly and outdoor connections are sealed with suitable waterproof tape During installation and commissioning it is important to conduct comprehensive tests to identify any sources of RF interference During maintenance visits, check cables and connectors for damage, cracking and water ingress – replace if necessary
Ionosphere – position bias in single frequency systems	Failure of iono-model to cancel out the effects of ionosphere delay in single frequency systems. Typically occurs during periods of increased ionosphere activity	Use GNSS receivers to calculate true ionosphere delay error and an augmentation service that can remove ionosphere delay error to cancel this effect
Ionosphere – scintillation	Causes rapid fluctuations in the phase and amplitude of the L-band satellite signal as it passes through small-scale irregularities in the ionosphere. This can cause the receiver to lose lock to the GNSS satellites and also the L-band augmentation satellite link. The effects of scintillation appear in different localised regions of the sky and thus only affect certain satellites at a time	If scintillation is detected it may be necessary to disable the particular satellite that is causing problem Ensure redundancy in delivery of augmentation data (i.e. from different satellites and/or terrestrial broadcast). When possible, use multiple DGPS reference stations and/or a PPP augmentation solution Consider use of multiple GNSS systems to provide additional satellites (e.g. GPS and GLONASS)
Loss of correction data	Failed correction data link. Failure of reference station(s)	Ensure redundant and diverse correction links Ensure reference stations have redundant equipment and communications links Use multiple reference stations, where possible, or use a PPP augmentation solution.

[6] IMO Resolutions 112, 113, 114 and 115 for the Performance Standards of Equipment and Annex D (GNSS) of IEC 61108-04

Effect	Cause	Mitigation
Change of reference station or station combination used affecting position accuracy and redundancy	Incorrect/inappropriate selection of reference stations or vessel/vehicle has moved to new work location	Ensure selection of appropriate reference stations for work location (DGPS) or use a PPP solution Changes to system configuration should be formally recorded (preferably in a change management system)
Poor satellite geometry or insufficient number of satellites	Elevation mask change	Ensure elevation mask is set to a value such that stable tracked GNSS satellites are available for the position solution (typically between 5° and 12°) Changes to system configuration should be formally recorded (preferably in a change management system)
	Satellite masking caused by obstructions	At mission planning stage check whether obstructions will impact positioning performance Consider reducing (if possible and without further degradation) the elevation mask to include additional satellites Consider use of multiple GNSS systems to provide additional satellites (e.g. GPS and GLONASS)
	Satellite de-selection – normally satellites are automatically flagged as unhealthy but occasionally the user may disable a particular satellite	Ensure that only problem satellites are de-selected and when problem has cleared satellites should be re-introduced Changes to system configuration should be formally recorded (preferably in a change management system)
	Changing constellation availability due to unhealthy satellites or satellite manoeuvres	Monitor system performance and official warnings if available for any signs of degradation Consider use of multiple GNSS systems to provide additional satellites (e.g. GPS and GLONASS)

Loss of GNSS signal	Intentional signal jamming can occur when certain bodies (e.g. regulatory or military) conduct jamming trials. Intentional signal jamming can also occur if military forces are operating near to work location or if the user is operating close to military installations	Regulatory bodies will normally issue notifications of where and when jamming trails are undertaken. Users should check notifications to see if work location is affected User should monitor systems if military forces or installations are nearby for any signs of degradation in positioning
	Un-intentional signal jamming is typically caused by RF interference from other transmitting devices – for example re-radiating GNSS, microwave transmission links on offshore platforms, and military radar	Conduct tests to discover source of interference by: systematically switching off transmitting devices Check antenna, cabling junction boxes for signs of damage, degradation or water ingress and repair if necessary If working close to installations (e.g. offshore platforms) check for any transmitting communications devices such as microwave links which can cause interference
Equipment failure	Failure of hardware including GNSS receivers, correction receiver, PCs. Issues with software not operation or suffering corruption. Damage or degradation in condition of antennas and cables	Ensure two redundant systems are installed and operational Have 100% spare equipment available including software installation files Ensure all configurations, where possible, are backed up Conduct regular inspection and maintenance to mitigate potential problems occurring

Table 4 – Factors influencing the performance of satellite-based positioning systems

4.5.1 Considerations for Different Installations

Static base station installations require careful planning to ensure effective GNSS system performance. Onshore static installations require the added consideration of hardware security, preventing the possibility of equipment being tampered with or removed. As with other installations, a suitable antenna mounting location is important to mitigate signal interference and multipath, and a location with a stable foundation should be selected. A multipath audit should be conducted and corrective action taken to ensure optimal performance. This may include the re-positioning of the GNSS antenna, removal of vegetation, fences etc. There should be periodic analysis to check that changing conditions have not caused degradation in performance.

Static and land surveying relies upon good, stable, signal reception and so it is necessary to plan survey sites and routes with satellite visibility in mind. Effective augmented GNSS system performance can be of a very high accuracy and care must taken when applying any offsets, including vertical offsets. Onshore static installations may require some security if left unattended. The recording and security of data should also be considered especially in remote areas where power may be limited. Re-occupation of control points, or key survey points, is strongly recommended to provide checks on the system performance.

Mobile offshore drilling units (MODU) offer particular challenges due to their tall structures (drill rigs, cranes etc.) and the height of the legs of jack-up rigs and barges. Moving structures such as cranes pose variable multipath difficulties. Further, if the MODU changes heading, the geometry of obstructions and multipath also changes. A solution to overcome the problems faced working with MODU is to employ two or more receiver systems with different antenna locations. This will help to ensure both GNSS and data link reception is maintained, regardless of the MODU's heading.

Vessel (ship) based activities are relatively straightforward so long as GNSS antennas are mounted high and clear on navigation masts or similar structures. Complexities may arise when a vessel is in close proximity to an obstructing structure which can introduce signal masking, e.g. a fixed platform, construction barge or MODU. The possible effects of such obstructions should be taken into account during mission planning. Fixed offsets should be applied in this dynamic environment to ensure that satellite-based positioning systems meet their specifications.

Small craft working is susceptible to obstructions restricting the sky view. Under calm sea conditions multipath effects may become apparent. A suitable ground plane on the antenna can reduce the effects of multipath. Small craft are often subject to relatively high dynamic movement which may shield the antenna. Also, rapid changes to a small craft's heading may cause loss of lock on the satellite signals. Their small size often limits the available space for positioning and backup systems.

Mobile vehicles can experience obstructions restricting the sky view, depending on the topography and infrastructure in the work area. Variable multipath is often experienced, caused by the dynamic observation environment. This can be mitigated by mounting an antenna with a suitable ground plane above the highest part of the vehicle. The effect of vehicle vibrations and RF (radio frequency) interference should be carefully considered during system installations.

Airborne activities are generally subject to certification and safety checks prior to their start. However, for positioning purposes aircraft, whether helicopter or fixed wing, are likely to maintain relatively smooth transitions between directions although there may be considerable acceleration. Special attention should be paid to the vertical component. Another aspect to consider is the rapid transit of an aircraft across an area that may test the coverage of a localised augmentation system. This is the component that may prove difficult to install as the augmentation data link may not be suitable for aircraft mounting. Relatively high data rates and high 3D accuracy will be required to maintain positioning integrity of the sensor data. Data recording is highly recommended to enable the post-processing of the positions. Space and weight restrictions in aircraft may limit the equipment carried for positioning systems and any backup units.

4.5.2 Risk Management

The issue of risk management of augmented GNSS services is complex. Occasionally, an unscheduled event may lead to performance degradation, signal interruption or loss of service. Occurrences may be infrequent and may only have a temporary impact upon the quality of positioning, making their detection and isolation difficult in real time. Fortunately many of these events are short-lived and with sufficient observations and quality measures, they can be identified and removed from datasets. Tables 5 and 6 outline some of the main elements within the service infrastructure and the user's location, the risks and some suggested mitigation measures. They give a relative indication of how likely an event is to occur and what the impact of that event may be.

Item	Comment	Risk: Probability of Occurrence	Risk: Severity	Possible Mitigation Measures
Communication satellite	Some service providers offer (on a commercial basis) a choice of broadcast satellites or a 'back-up' satellite covering the same world region. Communication satellite failure is rare	Low	High	• Plan a backup alternative with the same service provider • Use a different satellite data link • Use a different data link technology
GNSS reference stations	Most service providers maintain at least two 'hubs' with redundant equipment on 'hot' standby and automatic re-routing in the unlikely event of total failure	Low	Low	• Discuss with service provider to establish whether there is a single point of failure
PPP solutions	These solutions do not depend on reference stations *per se* but on a globally integrated network of GNSS tracking stations. These networks are particularly robust and can suffer circa 20% failure before any effect on accuracy is discernable	Low	Low	• Ensure there is no single point of failure
Position solution software corrections receiver	Where the software comes from a reputable supplier, the chances of errors are very low. However, software can become corrupted	Low	Moderate	• Have a standby package. • Include an independent, e.g. QC processing package • Carry spare system(s) • Have two receiver systems operational at all times
GNSS receiver	Stand-alone GNSS receivers can fail or become damaged	Moderate	Moderate	• Have at least two systems running at all times. Include an independent, e.g. QC processing package
Installation	The most common single points of failure causes are related to installation	High	Moderate	• Follow correct installation and checking procedures. • Install dual independent and redundant systems

Table 5 – GNSS system elements that may introduce risk and degrade positioning

Abnormal geomagnetic conditions can lead to heightened risk of failure or system degradation. The following table provides a means of assessing and remedying single points of failure under conditions of heightened solar activity (11-year solar cycle).

Item	**Comment**	**Probability of Occurrence**	**Severity**	**Possible Mitigation Measures**
Zones of increased risk	±15° of the geomagnetic equator and polar regions	High	High	Mitigation through the use of multiple systems with redundancy of data links, satellites and calculations
DGNSS reference stations	In times of increased solar activity one or more DGNSS reference stations can suffer interference or temporary loss	High	Moderate	Select a vendor offering at least two data link satellites using different stations covering the area OR engage a second vendor offering different reference stations and links OR use PPP and post process
GNSS receiver	In times of increased solar activity, single frequency GNSS receivers may experience interference, degraded signal or complete loss	Moderate	High	Use dual frequency receivers to mitigate the impact of ionospheric interference. In the event of complete loss of signal, this mitigation becomes redundant

Table 6 – Environmental influences on the performance of DGNSS

5 Quality Measures – Introduction to Quality Assessment and Statistical Testing

The aim of this section is to present a set of meaningful quality measures describing the ongoing statistical testing that must take place during position estimation from GNSS data. The quality measures and method described are generally referred to as the "Delft method" of quality assessment. The objective of the quality measures and statistical testing is to determine the quality and accuracy, and hence fitness for purpose, of the positioning. The methods are not only applicable to GNSS service providers for use at reference or monitor stations, but more importantly to all users processing augmented observations at mobile receivers. A general non-mathematical review of tests and measures is presented, including:

- precision measures;
- reliability measures;
- accuracy;
- statistical testing.

There is a list of quality measures and acceptance criteria recommended for use with GNSS indicating the method for implementing these into processing procedures. Appendix A provides a mathematical description including relevant derivations and explanatory examples. This section provides the necessary statistical background to the understanding of the chosen quality measures and gives specific guidelines for their use. It concludes with a recommendation that two quality measures be implemented for all offshore GNSS activities, namely:

- the 2D error ellipse or 3D error ellipsoid as a precision measure;
- the largest horizontal position vector resulting from a marginally detectable error as a measure of external reliability.

It should be noted that the specifics of RTK positioning, with its integer carrier ambiguity parameters, will not be dealt with here. The statistical properties of integer parameters are different from their real number equivalents. However, in practice, often the same quality measures are used for both types of parameters. It should be kept in mind that this is only allowed when there is high confidence that the ambiguities were properly fixed to their integer values.

In any measurement process, perfection is unattainable – errors will remain in all measurements, however sophisticated. Calibration and careful measuring procedures will further reduce these errors, but they cannot be completely eliminated. Computations with these imperfect measurements will in turn cause imperfections in the final calculated positions. These can be described in terms of an error distribution qualified with a certain confidence level. *Assessment of quality* in GNSS hence means assessment of the size and nature of the undetected errors in GNSS derived positions. Associated with quality assessment is a process known as *statistical testing*. Statistical testing is used to determine whether or not the assumptions made in the quality assessment process are correct.

The method used is an implementation of receiver autonomous integrity monitoring (RAIM). This is a technique used to assess each satellite signal, with the aid of the redundant satellites, for outliers that are inconsistent with the receiver position solution. Each satellite can be tested for agreement with the value for the receiver's position and if the difference exceeds a threshold the satellite may be removed from the solution.

5.1 Precision and Reliability

In GNSS operations, or in any other measurement activity, three kinds of errors are possible: random errors, systematic errors and gross errors. Appendix A describes the mathematics involved to determine the size of an error that can be detected and its effect on the estimated parameters if it is not detected.

- **Random errors** are by definition unpredictable. A perfectly random process is one in which an event is completely independent of other events. A good example is the throwing of dice – it is not possible to predict the throw of dice by knowing what has been thrown before. Random errors in measurement science are caused by small fluctuations in the physical factors that constitute the measurement process. In the case of GNSS they are due to factors such as scintillations in the atmosphere (the same effect which causes stars to 'twinkle') and electronic noise causing imperfect code cross-correlation. Random errors are described by statistics and consequently considered as stochastic quantities. In this document all random errors are assumed to belong to a normal distribution. This point is explained in a little more detail in section 5.3.

- **Systematic errors or biases** are any errors that are not described by the statistics used to describe the random errors. They can be completely predicted from a (not necessarily known) mathematical relationship and in practice are normally removed (or otherwise accounted for) by careful calibration and modelling. An ionospheric delay is an example of this type of error.
- **Gross errors (or outliers)** are similar to biases in that they are (often) large errors that do not belong to the distribution used to describe the random errors. They are typically caused by sudden changes in prevailing physical conditions, such as the acquisition of a new, very low elevation satellite, or the onset of multipath when passing close to a structure.

  Another form of gross error, which could be termed a blunder, is distinct from an outlier as it is not part of the measurement process. Blunders are often human errors, such as defining an incorrect offset or Co-ordinate Reference System. Statistical techniques cannot detect blunders. However, they can be identified with careful calibration and verification checks, as described in section 4.

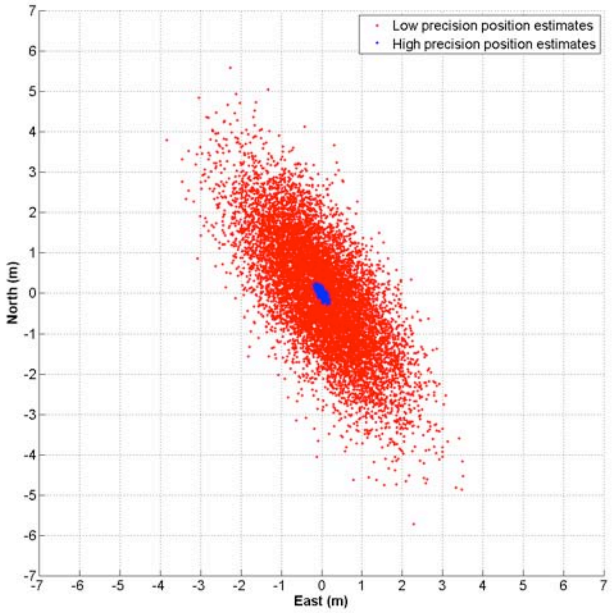

Figure 6 – High and low precision position estimates

The term *precision* is used to describe the quality of a GNSS fix with respect to *random errors*. A very precise fix is one in which the random errors are small and repeatability therefore is high. Such a fix is said to be of high precision. Conversely, fixes subject to large random errors are said to be of low precision. See Figure 6 for an example. Precision is assessed by describing the population from which the random errors are drawn. It is obvious that the errors themselves cannot be quantified (otherwise the observations would be corrected and the errors would no longer exist). In general a measure of precision would state the probability (chance) of there being an error of a certain size, or the size of error relating to a specified probability.

The term *reliability* is used to describe the quality of a GNSS fix with respect to outliers. A highly reliable fix is one in which even quite small outliers in the data will be noticed (detected), whereas in an unreliable fix large outliers will go unnoticed. Reliability is largely driven by *redundancy* – as the number of satellites (or reference stations in the case of DGNSS positioning) increases, so does the redundancy and hence the reliability. High *internal* reliability means that small measurement outliers can be detected (i.e. only very small outliers remain undetected) and high *external* reliability means that those measurement outliers that are not detected have very little effect on the final position estimate. In general, reliability is measured by stating the size of the error that might remain undetected in either the measurement domain (internal) or position estimate domain (external) with a specified probability.

5.2 Dilution of Precision

As GNSS satellites orbit the earth, a GNSS receiver is able to observe the effect of the relative receiver satellite geometry. The code signal from each GNSS satellite has a level of precision associated with it. The effect of geometry of the satellites on position error is called the dilution of precision (DOP) and it is interpreted as the ratio of position error to the range error. This relative geometry of the satellites and the signal precision can be combined to give a value of merit of the precision of the positioning solution. Importantly it can be predicted.

When the tracked GNSS satellites are near to one another in their orbits, the geometry is said to be weak and the DOP value high. When the satellites are well distributed in azimuth and elevation the geometry is said to be strong and the DOP value relatively low. Thus a low DOP value represents a better position precision due to the better angular separation between the satellites used to calculate a position. DOP can be expressed in a number of separate dimensions including PDOP (position) or GDOP (geometric); HDOP (horizontal); VDOP (vertical), and TDOP (time).

DOP values are calculated in the GNSS receiver or positioning computer system and are output in various data strings**.** Unfortunately the DOP value cannot predict the actual measurement errors encountered in real time. Consequently, alternative quality measures must be adopted and used.

5.3 Position Estimation

In order to convert GNSS pseudo-range or carrier phase observations into positions, two models are used: a functional model and a stochastic model.

- The **functional model** describes the mathematical relationship between the measurements and the required unknown position co-ordinates (see Appendix A);
- The **stochastic model** describes the statistical quality (i.e. precision and covariance) of the measurements.

The functional models typically used for GNSS are largely uncontroversial. A very simple one is given in Example 4 of Appendix A. Incorrect functional models can result in biases of the 'systematic error' type, also known as model errors, and would be seen during calibration. All GNSS service providers use virtually the same functional models in their software and they are not considered further here.

Stochastic modelling on the other hand is a difficult and controversial topic. It is extremely difficult to describe the precision of, for example, a differentially corrected carrier phase aided pseudo-range. Many factors influence this precision, including (amongst other things), the elevation angle of the satellite; the time since the satellite was acquired; atmospheric conditions; the distance to the reference stations; the latency of the differential corrections; the receiver noise characteristics and the multipath environments at both the reference and the mobile stations. Most GNSS system providers will have spent significant effort researching their own proprietary algorithms and they are unlikely to provide exact details of their procedures. It must, at the outset, be emphasised that the methods outlined in this document are based on the assumption that the adopted stochastic model is correct. Although a test to check this will be described (in Appendix A) this will only be able to check that the model is correct 'on average' – of course significant periods of failure might be interpreted as an error in the basic algorithm.

In general, precision of the GNSS observables, as captured by their standard deviation, does not only describe the random errors in the measurements, but also (residual) effects due to error sources that cannot be completely accounted for. Figure 7 shows the random noise of code observations, as obtained from zero baseline data (two receivers connected to the same antenna, effectively removing the biases, gross errors and outliers described in section 5.2) and the noise as obtained from a single receiver from the same antenna location. The noise is no longer random and caused mainly by multipath effects. In practice, the observation standard deviation is chosen such that it takes into account these remaining biases. Not doing this would result in too many rejections.

Once the two models are known, position fixes are usually computed by a process known as *least squares estimation*. This is a completely mechanical process, depending only on the two models. It can be shown that, based on certain not unreasonable assumptions, the least squares process gives the most 'desirable' result (i.e. the one of the highest precision) for any set of data. It is also, conveniently, relatively simple.

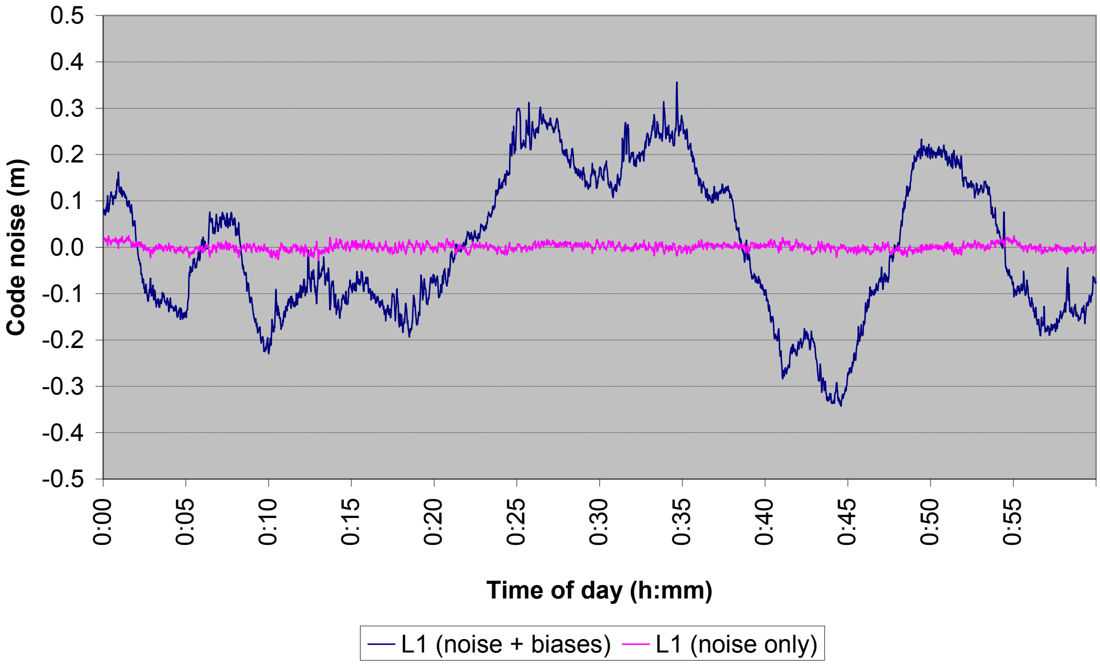

Figure 7 – Code observation noise, derived from a zero baseline (top) and from single receiver data (the same data as used in the zero baseline)

5.4 Measures of Precision

Precision is usually measured by means of a *standard deviation*. The terms *standard error* and *variance* may also be used (variance is the square of standard deviation). The standard deviation is a measure of the spread of the random errors remaining in any component of a position – the larger the standard deviation, the larger the random errors. Standard deviations of positions are determined by assessing the standard deviations of the measurements (and their correlation, if appropriate) and computing their propagation through the least squares process.

For GNSS observation processing, it might be useful to model the covariance between, for example, similar azimuths and elevations of a group of pseudo-ranges, to reflect the fact that they see similar atmospheric related delays. Another potential correlation scenario may be correlation between code and carrier due to applying the same orbit and clock corrections to both types of observations. There may also be time correlation if the sample rate becomes too high.

In the co-ordinate estimate domain, standard deviations are figures associated with individual variables, such as latitude and longitude. Rarely are the standard deviations uncorrelated in the reference frame of the survey. In order to depict uncorrelated standard deviations in two or three dimensions, error ellipses or ellipsoids are used and these are described later in this section.

In practice it is useful to assign probabilities to standard deviations, but this cannot be done without some assumptions regarding the nature of the population from which the errors are drawn. A population of errors can be described by a *probability density function* (PDF). This is simply a mathematical function which, when integrated, gives the probability of an individual error falling between specified bounds.

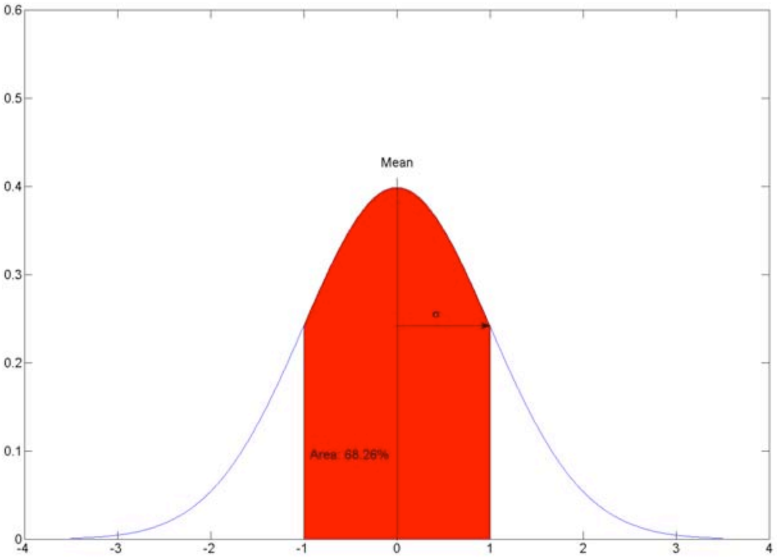

Figure 8 – One-dimensional normal distribution with mean equal to zero and standard deviation σ equal to 1

In practice truly random errors follow certain known rules and (so long as there are no biases present) it can be shown that they follow a *normal* PDF, which is centred at a parameter's mean or expected value, see Figure 8. The formula for the normal PDF includes the standard deviation σ and it is a straightforward matter to compute the probability of errors being within bounds of specified factors of the standard deviation. For instance, the chance of a one dimensional error of less than one standard deviation is 68.3% and the chance of an error being less than two standard deviations is 95.4%. Similarly it can be shown that there is a 95% chance that all one dimensional errors will be less than 1.96 standard deviations.

When considering horizontal (two-dimensional) positional standard errors for surveying it is not sufficient to look at only the one dimensional errors in the two main orthogonal directions of east-west and north-south. The standard error in the height component is sometimes disregarded as it is assumed that the height is well known. It is usually useful also to consider possible errors of position in all directions. For this, the *error ellipse* is used in two dimensions; for three dimensions an *error ellipsoid* can be used. An error ellipse is an approximate graphical representation of the standard deviation in two directions. The major axis of this ellipse lies in the direction of lowest precision (highest standard deviation) and conversely the minor axis shows the direction in which the fix is strongest. These directions do not necessarily coincide with the directions of the co-ordinate axes (e.g. north-south and east-west, see Figure 9). With respect to satellite geometry, the error ellipse becomes more circular when measurement availability in the direction of the semi-major direction is similar in quality and quantity to that in the semi-minor. Ideally, the ratio between major and minor axes should not exceed two. Note that the probability figure associated with a one sigma (standard deviation) ellipse is 39.4%, which is significantly smaller than the 68.26% associated with a 'one dimensional' standard deviation. The reason for this is that error ellipses are making statements about the precision of the position in two dimensions.

Error ellipse axes are commonly drawn at a scale 2.448 times their one sigma values and are referred to (correctly) as being 95% *confidence regions*, see again Figure 9. A proper, but not practically possible interpretation, is obtained by drawing such an ellipse centred at the true position – then it can be said that 95% of all measured positions should lie within this figure. In practice the ellipse is often drawn centred at the estimated (measured) position and sometimes it is then (incorrectly) said that there is a 95% chance that the true position is within this figure.

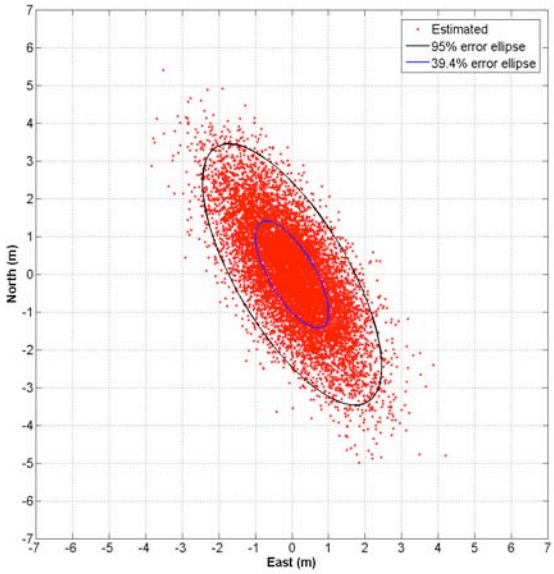

Figure 9 – Estimated 2D positions and error ellipses

Finally it should be emphasised that standard deviations and error ellipses are not measures of the actual errors – they simply describe the populations from which the errors come. They therefore should only change suddenly when that population changes, for example, if a new satellite is tracked. Otherwise they should change only very slowly – reflecting the gradual change in the geometry of the satellite constellation with respect to the receiver's antenna. This is not to say that certain test statistics (such as described in section 5.7) cannot vary in a random manner – indeed they should.

5.5 Measures of Reliability

Reliability is most usefully measured by means of *marginally detectable errors* (MDE). In order to understand MDEs it is first necessary to understand the basis of statistical testing, but before discussing this topic, it is once more emphasised that this guideline is concerned with *outliers* and consequently *reliability* is interpreted as the ability of a system to detect outliers and its sensitivity to undetected outliers.

5.5.1 Statistical Testing

When carrying out a statistical test for outliers a so-called *test statistic* is computed. The PDF of this statistic is known and if its value is so high that it can only to be expected to be exceeded in (say) 1% of cases, it is assumed that the observation contains a gross error and it is highlighted as a possible outlier (for probable rejection). The arbitrarily chosen percentage for this purpose (1% in the previous statement) is called level of significance (or level of confidence) of the test and is often denoted by α_0. In theory, α_0 percent of good data will be rejected (this is called a type one (1) error) but this is a price worth paying to be as sure as possible that bad data (outliers) are rejected.

Sometimes outliers may not be detected and data containing outliers will be accepted. When this occurs, a type two (2) error is said to have occurred and the probability of such an event is usually assigned the Greek letter β_0. Its complement is denoted by γ_0, i.e. $\gamma_0 = 1 - \beta_0$, and referred to as detection power (or power of test).

It is very important to understand that the choice of beta is of no real practical consequence – the choice only affects what we can say about the quality of the data – it does not affect what we actually do. This is in direct contrast to α_0. Varying α_0 will affect the amount of data we accept and therefore the results we obtain. However, in practice, and unless the data contains a very large number of outliers, any reasonable value, say, from 1% to 5% could be expected to lead to virtually identical results. Varying α_0 and β_0 directly affects our reliability statement – so whenever an MDE is quoted it will be essential to relate it to both α_0 and β_0. It is usually very simple to re-compute the MDE for a different α_0 and β_0, if required.

5.5.2 Internal Reliability

Internal reliability is directly measured by an observational MDE. Its meaning for GNSS applications is probably best explained by an example. It depends on satellite geometry, the type of GNSS observations, their standard deviations and the values for α_0 and β_0. Say that values of α_0 and β_0 are chosen to be 1% and 20%, respectively and observations consist of pseudo-ranges only. Satellite geometry (azimuth and elevation) and observation standard deviations σ (which are a function of satellite elevation) are given in Table 7. This table also gives the observational MDEs.

Satellite	Azimuth (°)	Elevation (°)	σ (m)	Observational MDE (m)
2	107	17	5.8	24.2
4	70	19	5.1	31.0
5	264	73	3.0	12.8
9	132	49	3.1	16.2
12	15	87	3.0	13.9
14	283	46	3.1	17.5
29	197	16	6.1	38.8
31	302	5	11.9	46.7

Table 7 – Satellite azimuth and elevation, standard deviation of pseudo-ranges and internal reliability (observational MDEs)

The following statement can then be made.

When outlier detection is carried out with a level of significance of 1% then there is a 20% chance of an outlier in the observation of satellite 9 of 16.2 metres remaining undetected.

Clearly if β_0 were increased, exactly the same data would be accepted or rejected, but the statement would now include a number less than 16.2 metres (and vice versa). For any given α_0 and β_0, the MDE is a direct measure of internal reliability. The larger the number, the less reliable the position fix (good fixes can detect small outliers).

5.5.3 External Reliability

External reliability is actually a more useful concept in practice, because it might be that, in a particular fix, a large undetected outlier may have little effect on the horizontal position fix (for instance, if it is related to a very low weighted satellite). External reliability is assessed by the largest positional MDE, see A.42 in Appendix A. It is computed by first computing the MDE for each observation and then propagating the effect of each MDE through the least squares process to the final 3D position. This will result in a 3D bias vector; the length of this vector is the positional

MDE. Each observational MDE will cause a different positional MDE and it is the largest effect (i.e. worst case) that is quoted (along with a statement identifying the relevant measurement).

Consider the satellite constellation and observation standard deviations of the example of section 5.5.2. Table 8 is expanded with positional MDEs.

Satellite	Azimuth (°)	Elevation (°)	σ (m)	Observational MDE (m)	Positional 3D MDE (m)
2	107	17	5.8	24.2	5.9
4	70	19	5.1	31.0	33.1
5	264	73	3.0	12.8	11.1
9	132	49	3.1	16.2	45.4
12	15	87	3.0	13.9	42.9
14	283	46	3.1	17.5	26.7
29	197	16	6.1	38.8	13.0
31	302	5	11.9	46.7	6.7

Table 8 – Satellite azimuth and elevation, standard deviation of pseudo-ranges and internal (observational MDEs) and external (3D positional MDEs) reliability

The following statement can be made:

When outlier detection is carried out with a level of significance of 1% then outliers can still remain undetected in any of the measurements. The occurrence of an undetected outlier in the measurement to satellite 9 would have a more detrimental effect on the final position than such an occurrence in any other measurement, and if an MDE occurred at the 20% level of detection in this measurement, it would cause an error of 45.4 metres in the final position.

External reliability is concerned solely with the effect of *undetected* outliers on the final positions. We cannot make any statements about the chances of outliers occurring – we can only discuss the possibility of detecting them if they do occur, and their effect if we fail to detect them. This is what is being assessed by reliability.

5.6 Accuracy

Precision was defined as a measure of the random errors in observations and estimated parameters. Reliability is the ability to detect outliers in observations and the impact of undetected outliers on the estimated parameters.

Precision \ Reliability	Good	Bad
Good	High accuracy	Low accuracy
Bad	Low accuracy	Low accuracy

Table 9 – Accuracy and its relationship with precision and reliability

Accuracy can be considered as a combination of the two. A high accuracy means high precision and high relaibility, the ability to detect small outliers in observations and a small impact of undetected outliers on the parameters of interest, i.e. the estimated positions. Table 9 gives a brief overview. Gross errors remain a possibility and regular checks for gross errors should be carried out.

5.7 Specific Statistical Tests

It is recommended that two tests be carried out during each position fix computation. The details of the two test statistics, their PDFs and testing procedures are described in Appendix A. The purpose of this section is to explain the goal and interpretation of the tests.

Firstly it must be emphasised that statistical testing is not formally part of the quality assessment process in the sense that it does not lead to a number that can be quoted to describe quality (as is the case of standard deviations for precision and MDEs for reliability). *Test statistics are not quality measures.*

The purpose of statistical testing is to confirm that the functional and stochastic models used to compute the precision and reliability measures are indeed correct. Often these tests from one individual fix are not sufficient for this and averages over large number of epochs need to be considered. It is beyond the scope of this document to deal with this point in detail – it is sufficient to comment that good quality GNSS software should have a means of feeding the *filtered values* of test statistics back into the models (either manually or automatically). Individual values have no role in this process – their use for this purpose would cause quality measures to change rapidly and it has already been explained in section 5.4 why this should not happen.

Within least squares all statistical testing is based on the *residuals*. Note that residuals are simply the amounts that need to be added to the measurements to make them geometrically consistent with the least squares estimate of the position, i.e. they are the *corrections* to the measurements. Another way to think about residuals is to consider them to be estimates of the *measurement errors* (with a change of sign).

5.7.1 w-Test

The w-test (or local slippage test, see Appendix A, equation A.39) is used to identify outliers in the data and, given that the long term use of the unit variance has indicated that the models are correct, the usual action would be to reject the measurement concerned and repeat the least squares estimation of the position.

In the event of more than one outlier being detected, the usual procedure would be to reject only the measurement with the largest (in absolute sense) w-test statistic and then to repeat the computation. It might be that all measurements now pass the test, but if not, the process can be repeated.

Essentially the w-test statistic (in the usual case of uncorrelated data) is obtained by dividing the residual by its standard deviation; see Example 9 in Appendix A. It is often referred to as the *normalised residual*. Since, for a normal distribution, 99% of such results should be less than 2.576, it is assumed that any value larger than this is very likely to have been caused by making a measurement with errors from another population, i.e. an outlier.

Finally, it should be noted that, in principle, measurements should not be rejected without first investigating possible reasons for them being outliers. In practice, especially in highly automated and complex systems, this investigation is unlikely to be possible in real time and there is little option but automatic rejection. It is nevertheless recommended that a log of rejected data be kept for later investigation.

5.7.2 Unit Variance Test

The *unit variance* statistic is computed from equation A.36 in Appendix A, where it can be seen that (in the common case of uncorrelated measurements) its value is largely driven by the weighted sum of squares of the residuals. It can be shown that the statistical expectation of the unit variance statistic is unity and for this reason, individual values of the unit variance are tested against unity.

This does not mean that any individual unit variance should be exactly unity – merely that on average this should be so. It is, however, the case that very large individual values of the unit variance are not expected. The results of unit variance testing can be interpreted as follows:

- Occasional very small values of the unit variance are not a cause for concern. The unit variance can even be zero – this just means that the measurements for the fix in question happen to fit together perfectly.
- Occasional large values (that fail the test) indicate outlier(s) in the data – which would have been identified by the w-test.
- Long-term average significantly greater or less than one indicates model errors. Tests to identify which kind of model error are beyond the scope of this document, but if residuals are consistently the same sign or if they exhibit other obvious patterns, there is probably a problem with the functional model. If the residuals are random, the problem will lie with the stochastic model.

The importance of this test cannot be over-emphasised. If the models are not representative, i.e if the long-term average unit variance is significantly different from unity, the results of the precision and reliability assessment cannot be relied upon and the model parameters should be investigated.

5.8 Recommendations for a Typical GNSS Survey

In the previous sections the concepts underlying the rigorous measurement of quality of positioning in terms of both precision and reliability were described. In Appendix A, these concepts are given in mathematical terms.

This section focuses on guidance on the mobile/dynamic use of GNSS services utilising augmentation data. As such it excludes a detailed discussion of the RTK positioning technique used for land and near-shore survey. As the nature of this work calls for compact/hand-held receivers, it often precludes the use of external QC software. RTK receivers should provide comprehensive quality information with which to assess the precision and reliability of the position solution. The following is a brief overview but for a more detailed discussion the reader should refer to RICS Guidance Notes for further information.[6]

The purpose of this section is to provide guidance to users on typical acceptance criteria when GNSS services are utilised for positioning. It is assumed that the position computation is based upon the principle of least squares and some form of continually filtered, online network adjustment process. Broadly speaking, the computer to which GNSS and all other requisite positioning sensors are interfaced should therefore operate along the lines mapped out in the flowchart in Figure 11 and should be capable of outputting quality control diagnostic information similar to that shown in Figure 10. The presentation format, most of which is likely to be graphical, will vary depending upon contractor and supplier and will be a matter of user preference.

In order to carry out rigorous QC, the covariance matrix and residuals generated by the least squares computation should be used to generate **test statistics** and **quality measures**. Figure 10 gives examples of some of these quantities. GPS ionosphere-free code observations were processed in kinematic mode (one new position for each observation epoch) for a typical day. Shown are overall test statistics, 3D positional MDEs and standard deviations for each position component for the entire day. Also shown are w-test statistics for one satellite pass of about five hours. From this figure, it is clear that the overall test statistics vary rapidly and that their average value, for this particular case, is not equal to one. It is actually smaller, indicating too pessimistic values for the stochastic model of the observations. As can be seen from this figure and in more detail from Table 10, a high precision does not necessarily correspond to a high reliability. Table 10 also shows that an extra satellite could have a dramatic impact on the 3D MDE, whereas the effect on precision is much smaller. Note that the w-statistic is correlated with the elevation angle. Ideally the stochastic model should dynamically change to reflect the increased noise, in this case presumably due to decreasing elevation angle and accompanying ionospheric delay. This would also result in overall test statistics that are closer to the expected value of one.

[6] RICS Guidance Note ISBN 1 842219 093 8 *Guidelines for the use of GPS in Surveying and Mapping*

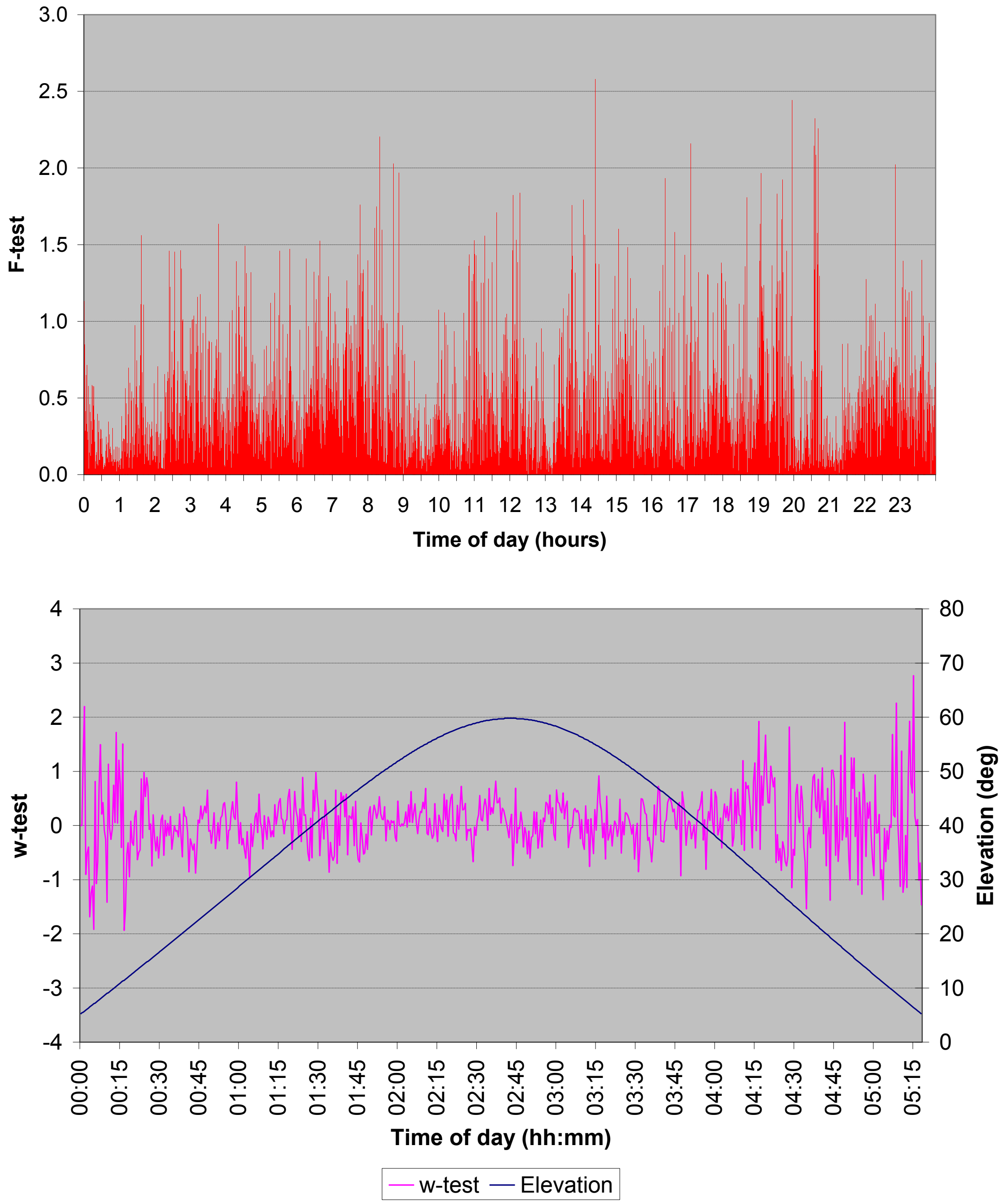

Figure 10 – Overall test statistics for an entire day (top) and w-test statistics and elevation for one satellite pass of about five hours, for a code only, ionosphere-free, kinematic GPS solution

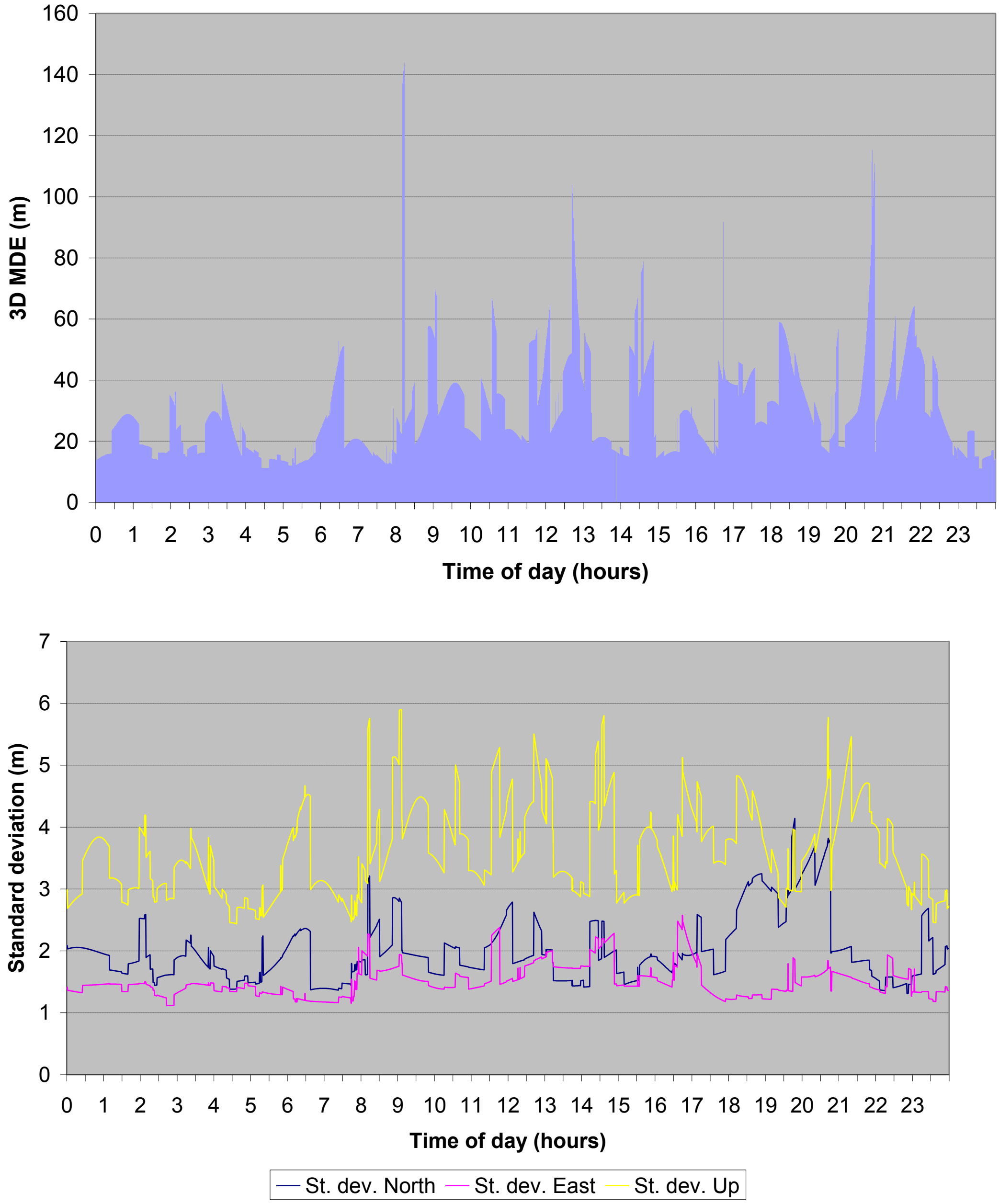

Figure 11 – 3D positional MDEs (top) and standard deviations of estimated position components for an entire day for a code only, ionosphere-free, kinematic GPS solution

Time	Standard deviation			3D MDE	No. of satellites
	North	East	Up		
8:14:00	3.2 m	2.3 m	5.8 m	143.8 m	8
8:14:30	2.2 m	1.6 m	3.4 m	25.8 m	9

Table 10 – Precision and external reliability for two epochs of data from Figure 10

5.8.1 Test Statistics

The recommended test statistics are the **w-test** and the **F-test** (unit variance test):

w-test: used to detect outliers. Observation residuals for which the magnitude of the w-test statistic, see Appendix A (A.37), is greater than 2.576 are highlighted for each position fix. If more than one outlying observation is highlighted in a fix, then only the largest should be rejected and the computations repeated, see Figure 11.

Acceptance criteria would therefore require that the mean w-test should have an average value that depending on the number of samples (i.e. statistical significance) is equivalent to zero over a period of time (e.g. one seismic line).

F-test: used to verify the model which is being used to account for 'errors' in the GNSS observations (e.g. atmospheric refraction, multipath, differential corrections). This is done by verifying that the average value of the unit variance is one. Again, it is not true that each computation yielding a value not equal to one is a bad one.

Acceptance criteria would, therefore, require that the mean of the unit variance computed by the F-test for a period of time (e.g. one seismic line) should be statistically equivalent to one.

These test statistics should be used to continuously monitor the quality of the GNSS measurements.

5.8.2 Quality Measures

The quality measures which should be computed and examined for each fix are the **error ellipse** and **external reliability** (3D positional marginally detectable error):

- Error ellipse: an approximate graphical representation of the positional precision in two dimensions. It should be used to indicate the size of random errors in the position and also the direction in which the errors are occurring, see also Figure 11.

When the error ellipse is drawn at a confidence level of 95% then there is 95% chance that the estimated position lies within the ellipse which is centred at the true position.

- External reliability: the largest effect on the estimated position of an observational MDE. The positional MDEs for each observation are a means of describing how reliable it is (i.e. how well it can be checked by other observations). MDEs should be computed for each observation (pseudo-range or carrier phase) included in the computation. It is recommended to use a significance level α_0 of 1% and a detection power $\gamma_0 = 1 - \beta_0$ of 80%. An example is given in Table 11.

This means there is a 20% chance that a gross error less than or equal to the stated value of the observational MDE will remain undetected.

MDEs are derived for each observation. External reliability is the greatest effect on position of an MDE (not necessarily the largest observational MDE, see Example 11 in Appendix A). This is a more relevant quality measure for a final position than the observational MDE. The largest observational MDE could, for example, be for a low elevation satellite which is weighted out in the final solution.

The important thing to remember is that reliability is not a measure of the errors in a solution, but of the likelihood that errors will be detected if they are present.

5.8.3 Summary

Table 11 gives a summary of the recommended parameters for assessing the quality of GNSS position fixes. The method by which the quality measures should be implemented into processing procedures is shown in Figure 12. The testing parameters in this table are based on Baarda's B-method, described in Appendix A. Therefore, no critical value for the F-test (or unit variance) is given.

Note that the test statistics and quality measures described in the previous sections are independent of the GNSS positioning method – they are equally valid for DGNSS, PPP, RTK (Real Time Kinematic) and stand-alone positioning, single-frequency and measurements, as well as code and carrier observations.

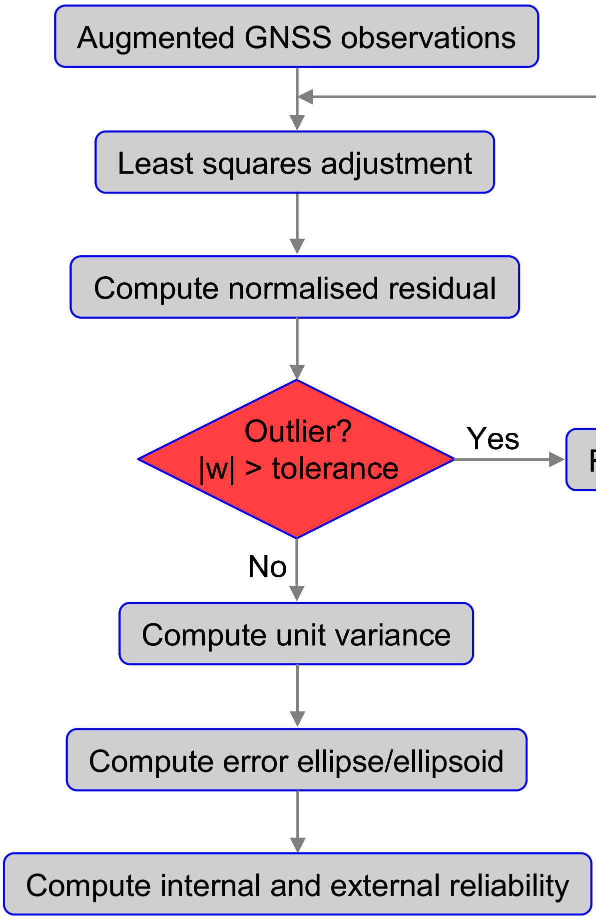

Figure 12 – Data processing flowchart

Measures	Effect	Recommended Value
Level of significance (α_0)	• Probability of rejecting a valid observation • Size of internal and external reliability measures	1%
Detection power ($\gamma_0 = 1 - \beta_0$)	• Probability of rejecting an invalid observation • Size of internal and external reliability measures	80%
F- test	Acceptance or rejection criterion (unit variance) for full functional and stochastic model	n/a
Critical value w-test	Acceptance or rejection criterion for a single observation	2.576 (99%)
Multiplication factor, 1D	Scale standard error ellipse to desired confidence region	1.96 (95% region)
Multiplication factor, 2D	Scale standard error ellipse to desired confidence region	2.448 (95% region)
Multiplication factor, 3D	Scale standard error ellipsoid to desired confidence region	2.796 (95% region)
Ratio major and minor axis	Isotropy of 2D solution	< 2*
Marginally detectable error (MDE)	Effect on 3D position of the minimum error that can just be detected in an observation with a given level of significance and detection power	n/a

* under normal operating conditions, dependent upon geographic location

Table 11 – Recommended parameters to assess the quality of GNSS position fixes

6 Competence

Personnel installing and operating GNSS should be suitably qualified. This section describes the recommended competences of such personnel.

Responsibility for personnel competences typically lies with the employer, e.g. survey contractors and GNSS service providers. Clients should take an interest in ensuring that the personnel responsible for GNSS installation and operation are suitably qualified and trained.

6.1 Recommended Competencies

To ensure appropriate and effective GNSS use and to ensure that systems are properly maintained it is recommended that personnel responsible for system selection installation and operation have the appropriate qualifications, training and experience to meet the competency requirements outlined below. In general, formal qualifications in geomatics or related disciplines, system specific and other training as well as practical experience, will be necessary to meet the competency requirements. Operator personnel with extensive field experience, who may not have had any formal training, may be considered competent if they can clearly demonstrate the required knowledge and practical performance.

Sufficient competence in key roles will enable operators, system providers and clients to manage the operation of GNSS such that the risk of adversely affecting system performance through incorrect or inadequate use is minimised and should result in optimal GNSS system performance through efficient operation and system control.

6.2 Competence – Knowledge

Competence should be based on a basic understanding of the theory of GNSS. Knowledge of system configuration and operation of GNSS equipment, together with identification and understanding of potential error sources are required for practical problem solving.

A competent person should have fundamental knowledge and understanding of the following areas of GNSS application:

- basic GNSS principles – theory; control, space and user segments, signals, and system structure;
- GNSS position determination; pseudo-range and carrier phase observables, error sources and mitigation;
- GNSS augmentation; systems principles and practical application;
- geodetic reference systems; GNSS geodesy, satellite and local geodetic reference systems, geoid, ellipsoids, co-ordinate transformation;
- system installation, error detection recognition and avoidance;
- system operation; mission planning/coverage of satellites, data formats, firmware upgrade policies, bulletin boards;
- quality control; statistics, quality measures, least squares adjustment, error sources, QC and operating procedures, system integrity monitoring.

Due to the dynamic nature of GNSS technology it is important that operating personnel have recent practical experience with the type of system being employed. If no recent practical experience is available, appropriate training should be undertaken to ensure that the field operators are able to use the equipment effectively and in compliance with manufacturers' recommendations.

6.3 Competence – Performance

With the fundamental knowledge outlined above, the following tasks should be within the capability of competent GNSS operator personnel.

- identification of all hardware components and check on status of equipment once installed. This is to be achieved by understanding tests for signal integrity, signal to noise, and receiver settings, including output formats, system parameter settings and recording formats;
- selection of a location and installation of a GNSS antenna (where applicable) in the best available location;
- installation of GNSS reference/monitor and mobile stations, giving due regard to obstructions, signal to noise levels, requirement for antenna amplifiers, interference and multipath sources;
- correct selection and configuration of the relevant correction and GNSS data formats in the DGNSS computer. Due regard should be given to any co-ordinate transformations that may be required;
- operation of the augmented GNSS system at optimum performance;
- operators should be sufficiently conversant with a system to modify its operation when required and keep track of changes made to the system for handover;
- monitoring and maintenance of system performance through consideration of positioning quality measures, system bulletin communications, firmware upgrade and strategic planning;
- understanding, planning and performance of appropriate system verification/calibration when required;
- integration of peripheral devices to the DGNSS system/computer for timing or positioning reference, with due consideration for timing convention, data format and data latency issues;
- accurate reporting, log-keeping and data management, such that data is stored, archived and backed-up in a logical and systematic manner.

The demonstration of competence in the aspects outlined above generally relates to the installation and maintenance of a GNSS system. However, the requirement for sufficiently trained/experienced personnel in data processing is just as important. In most GNSS applications, at least in a dynamic environment like marine surveying and positioning, it is the real time determination of position that is critical for operations. Data processing, particularly post-processing is seldom considered as important however, competent personnel with the necessary technical knowledge are also required for GNSS data processing, particularly in the static terrestrial environment, or for PPP used as verification checks, in which post-processing is more common.

7 Receiver Outputs/Data Exchange Formats

This section briefly describes the GNSS receiver data output formats and various oil industry positioning data exchange formats that are used to record raw and processed GNSS and derived positioning data.

In a surveying and positioning context the output from GNSS positioning systems may be regarded as the means to determining the positions both in real time and off line of survey sensors and points of interest.

The introduction of new and more sophisticated GNSS positioning techniques and of rigorous quality control based on statistical testing means that the traditional concept of a 'receiver' supplying a position is no longer appropriate. The GNSS receiver serves as the data source, and the position solution and associated quality measures are derived by separate software that for dynamic surveying and positioning typically run on computer hardware interfaced to the GNSS receiver or on land and other static surveys on a separate off line computer. Hardware configuration and software solution varies with e.g. type of solution, service provider etc. When discussing 'receiver' outputs, it may frequently be the case that the appropriate data will be made available by the software package rather than the GNSS receiver. The term *data output* is used in this section.

The objective of recording of GNSS data generally falls into two categories depending on the purpose of the surveying and positioning. On dynamic vessels and vehicles where real time positioning is necessary, raw GNSS data are typically recorded online as a back-up to ensure that re-processing of the data can take place if the real time positions are later found to be adversely affected by errors. On land and other static surveys GNSS data should be recorded to derive the positions of the surveyed points through post-processing techniques. Additionally raw GNSS data also needs to be recorded during equipment installation for testing and evaluation purposes.

In order to record, archive and exchange the calculated position data a number of position data exchange formats have been developed by the oil industry; these were primarily developed for exchange of seismic positioning data.

Use of a survey grade GNSS receiver should ensure that the user has access to four classes of GNSS data:

- raw observables with relevant satellite ephemeris and navigation messages;
- calculated position information with associated quality measures;
- precise timing information if relevant;
- GNSS based motion and attitude outputs if relevant;

7.1 Raw GNSS Observables

To allow the recording and exchange of raw GNSS data, the Receiver Independent Exchange Format (RINEX) was developed by the user community. The first proposal for the format was developed by the Astronomical Institute of the University of Berne for the EUREF 1989 GPS campaign.

The RINEX format supports only the recording of raw GNSS observables and satellite navigation information, and has traditionally been associated with largely onshore geodetic survey work. Although most receivers may record data in a proprietary format, most will also supply software to transcribe such data into the RINEX format. In addition the format is supported by most GPS processing software.

7.2 Calculated GNSS Position Information

7.2.1 NMEA Format

The National Marine Electronic Association (NMEA) standard is the most commonly used standard by manufacturers of GNSS equipment to output calculated GNSS position information. The standard defines the interface between various pieces of marine electronic equipment. GNSS receiver communication is defined within the standard. There are standard sentences for each device category and in addition NMEA permits hardware manufacturers to define their own proprietary sentences for their own use.

The NMEA standard does not itself include quality measures, and although GNSS service providers and software vendors may have introduced these parameters into their NMEA data output strings, not all integrated positioning systems are capable of acting upon any changes in these parameters. Users should consult with system providers to determine to what extent specific quality measures are used.

7.2.2 'P' Formats

For geophysical data acquisition operations the storage of all the necessary raw and processed positioning data can generate large data volumes. A standard approach for the recoding and exchange of seismic positioning data was developed by the former UK Offshore Operators Association (UKOOA), now Oil & Gas UK (OGUK), namely the P1 and P2 formats. These formats are designed to store all the relevant data for positions and enable the exchange through their standard header information and content designs. The P2 format was developed specifically for storage and exchange of raw marine seismic positioning data and the P1 format for the processed source and receiver positions.

The P2 format allows the recording of GNSS calculated positions. In addition, traditional differential corrections, as defined by the Radio Technical Commission for Maritime Services (RTCM) standard are also recorded. The P2 format also has provision for recording a limited set of quality parameters; however the format does not currently support recording all the raw GNSS observables and other satellite ephemeris data.

Custodianship of the UKOOA P formats has passed to the OGP Geomatics Committee, which is conducting a review of the P1 and P2 formats at the time of writing (2011). It is expected that the P2 format will be extended to include all of the recommended quality measures and allow recording of all raw GNSS data.

7.2.3 SPS Format

The SPS format was originally published by Shell and adopted by the Society of Exploration Geophysicists (SEG) Technical Standards Committee in 1993. It was originally designed for transfer of positioning and geophysical support data from land seismic surveys and consists of three main file types: receiver and source point files containing the co-ordinates and elevation of receiver and source points, and a relation or cross-reference file specifying the relation between recording channels and receiver groups.

7.3 Precise Timing Outputs

It is possible to obtain precise timing information from a GNSS receiver configured with the appropriate output ports. Survey equipment should be accurately synchronised to a common time frame to ensure that data integration is achieved without introducing timing latency errors. A GNSS receiver equipped with a 1(one) pulse-per-second (1-PPS) output and an appropriate time tag may be used to synchronise other equipment to the GNSS time. There is an offset between GPS time and UTC which must be applied in order that all components of the integrated survey system operate within the UTC time frame. In order for this synchronisation to be achieved the external equipment must be suitably configured to accept and use the PPS timing information.

The user should be aware of the relationship between the 1-PPS pulse and the associated time tag from the GNSS receiver; the tag may precede or follow the pulse.

Very high timing accuracy requires specialist GNSS receivers that have been developed to produce dedicated high accuracy timing rather than positioning. These units usually offer several timing output formats such as the standard I-PPS and the IRIG-B time code. Data rates to match the accuracy levels result in outputs of up to 10MHz.

7.4 Motion and Attitude Outputs

Some GNSS positioning systems also produce a real time motion or attitude output for offshore and other dynamic survey users to adopt as a reference for other sensors. These systems may comprise GNSS receivers with multiple antennas, but more often they are an integrated system with GNSS receivers coupled with an inertial measurement unit (IMU) or a motion reference unit (MRU). This is specialist technology usually requiring the special siting of multiple GNSS antennas and the IMU/MRU. The systems are capable of high data output rates, typically at 20-100 Hertz, required to compensate for the motion of a mobile platform.

The motion and attitude output from such systems is typically provided in proprietary manufacturer formats. The position component output is typically formatted in an NMEA standard; however the data elements are an integration of the GNSS observables, aided by additional information such as accelerations from the IMU/MRU.

8 Further Reading

8.1 Section 3

- *GNSS – Global navigation satellite systems*
  B Hofmann-Wellenhof, H Lichtenegger, E Wasle
  SpringerWienNewYork, 2007
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- *Bernese GPS software version 5.0*
  Dach, R, Hugentobler, U, Fridez, P, Meindle, M
  Astronomical Institute, University of Bern, January 2007 (Zero-difference reference)
  http://www.bernese.inibe.ch/docs/DOCU50.pdf
- *GPS satellite surveying*
  A Leick
  Wiley, 2003
  ISBN 978-0471059301
- *Inside GNSS*
  Magazine published eight times a year by Gibbons Media & Research LLC
  www.insidegnss.com
- *Manual on hydrography: publication M-13*
  International Hydrographic Organisation, May 2005
  www.iho.shom.fr
- *The journal of navigation*
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  www.journals.cambridge.org
  www.rin.org.uk
- *Cost effective GNSS positioning techniques: FIG publication 49*
  International Federation of Surveyors (FIG), 2010
  ISBN 978-87-90907-79-2

8.2 Section 4

- *Hydrography for the surveyor and engineer, 3rd edition*
  A Ingham, V Abbott
  Wiley-Blackwell, 1992
  ISBN 978-0632029433
- *Handbook of offshore surveying, Volume 1*
  Clarkson Research Services Limited, 2006
  ISBN 1-902157-73-7
  www.crsl.com
- Datums and map projections: for remote sensing, GIS and surveying, 2nd edition
  Whittles Publishing, 2008
  ISBN 978-1904445470
- IMCA M 199 – *Guidelines on installation and maintenance of DGNSS-based positioning systems*
  International Marine Contractors Association, August 2009
  http://www.imca-int.com/divisions/marine/publications/imca.html

8.3 Section 5

- *A baseline RAIM scheme and a note on the equivalence of three RAIM methods*
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VSSD, 2002
ISBN 978-90-407-2359-9

- *Principles of error theory and cartographic applications*
  CR Greenwall, ME Shultz
  ACIC Technology Report No. 96, 1968
- *Marine Positioning Multiple Multipath Error Detection.*
  S Ryan, G Lachapelle
  The Hydrographic Journal, No. 100, pp 3-11, 2001
- *Adjustment theory – an introduction*
  PJG Teunissen
  VSSD, 2000
  ISBN 978-90-407-1974-5
- *Testing theory – an introduction*
  PJG Teunissen
  VSSD, 2000
  ISBN 978-90-407-1976-2
- *Differential GPS : concepts and quality control*
  National Institute of Navigation (NIN) Workshop, Navstar GPS, Amsterdam, 27 September 1991
  PJG Teunissen
  http://enterprise.lr.tudelft.nl/publications/files/1991-007.pdf
- *Quality control in positioning*
  CCJM Tiberius
  The Hydrographic Journal, No. 90, pp 3-8, 1998

8.4 Section 6

- IMCA C 004 Rev. 2 – *Competence assurance and assessment – guidance document and competence tables: Offshore Survey Division*
  International Marine Contractors Association, November 2009
  http://www.imca-int.com/core/ct/publications/

8.5 Section 7

- The SPS Land Seismic data exchange file format:
  - http://www.seg.org/SEGportalWEBproject/prod/SEG-Publications/Pub-Technical-Standards/Documents/seg_sps_rev0.doc

8.6 Additional Resources

- The International GNSS Service (IGS)
  - http://www.igs.org
- NAVSTAR Global Positioning System Interface Specification IS-GPS-200D
  - http://www.navcen.uscg.gov/gps/geninfo/IS-GPS-200D.pdf
- European Space Agency – Galileo
  - http://www.esa.int/esaNA/galileo.html
  - http://www.gsa.europa.eu/
- Russian Space Agency – GLONASS
  - http://www.glonass-ianc.rsa.ru/pls/htmldb/f?p=202:1:9939630416051479874
- Chinese Beidou Satellite System (COMPASS)
  - http://www.beidou.gov.cn/ (in Chinese)
- United States Air Force (GPS)
  - http://www.schriever.af.mil/gps/index.asp
  - https://gps.afspc.af.mil/index.html

Appendix A – Estimation & Quality Control

Least Squares Estimation

Assume we want to determine n parameters, contained in the *parameter* or *state vector* x and that we have m observations available in the measurement vector y. If there is a known linear relationship between parameters and observations, we may set up the *measurement model* or *model of observation equations*

$$y = Ax \tag{A.1}$$

where A is the $m \times n$ *design matrix*. Here we will assume that $m \geq n$ and that the rank (the number of linearly independent columns) of A is equal to n. A solution to this linear system exists if the vector y can be written as a linear combination of the columns of matrix A. In that case (A.1) is called a *consistent* system; otherwise, the system is *inconsistent*.

Example 1 – Inconsistent system

Assume we have five measurements of the same quantity x, contained in the observation vector y:

$$y = \begin{pmatrix} 9.99 \\ 10.03 \\ 9.98 \\ 9.99 \\ 10.01 \end{pmatrix}$$

Since we assume that each measurement is a direct observation of x, we have

$$y = \begin{pmatrix} 9.99 \\ 10.03 \\ 9.98 \\ 9.99 \\ 10.01 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} x = Ax$$

For this example, $m = 5$ and $n = 1$. The above system is inconsistent as y cannot be expressed as a linear combination of the single column of matrix A. In practice, in order to get a single (unique) estimate of x, we usually take the average of the five observations

$$\hat{x} = \frac{1}{5}(9.99 + 10.03 + 9.98 + 9.99 + 10.01) = 10$$

Inconsistent systems can be made consistent by introducing an m-vector e in the model (A.1)

$$y = Ax + e \tag{A.2}$$

The *least squares* principle is based on minimising the discrepancy between y and Ax. This leads to the minimisation problem

$$\underset{x}{\text{minimise}} \|\mathbf{e}\|^2 \Leftrightarrow \underset{x}{\text{minimise}} (\mathbf{y} - A\mathbf{x})^T (\mathbf{y} - A\mathbf{x}) \tag{A.3}$$

A solution to (A.3) exists if

$$\frac{\partial\|e\|^2}{\partial x} = 0 \text{ and } \frac{\partial^2\|e\|^2}{\partial x^2} > 0$$

Expanding the squared norm in (A.3) results in

$$\|e\|^2 = (y - Ax)^T (y - Ax) = y^T y - 2x^T A^T y + x^T A^T Ax \tag{A.4}$$

Its first and second derivative with respect to x are

$$\frac{\partial\|e\|^2}{\partial x} = -2A^T y + 2A^T Ax \tag{A.5}$$

$$\frac{\partial^2\|e\|^2}{\partial x^2} = 2A^T A \tag{A.6}$$

The matrix on the right-hand side of (A.6) is positive definite, $\partial^2\|e\|^2 / \partial x^2 > 0$. This can be seen as follows: let z be an arbitrary non-zero m -vector. The matrix $A^T A$ is positive definite if $z^T A^T Az > 0$. Since Az is a vector, $z^T A^T Az = \|Az\|^2 > 0$ by definition.

Expression (A.5) is zero if

$$A^T Ax = A^T y \tag{A.7}$$

from which it follows that the estimator $\hat{x}$, the least squares solution to (A.2), is given by

$$\hat{x} = (A^T A)^{-1} A^T y \tag{A.8}$$

The vector of adjusted observations $\hat{y}$ is defined as

$$\hat{y} = A\hat{x} = A(A^T A)^{-1} A^T y \tag{A.9}$$

This vector is an element of the range space of A. The vector of *least squares residuals* $\hat{e}$ is defined as

$$\hat{e} = y - \hat{y} = y - A\hat{x} \tag{A.10}$$

Note that $A^T \hat{e} = 0$, i.e. $\hat{e}$ is orthogonal to the columns of A. In the above derivations, it was assumed that all observations have equal weight. Introducing the positive definite matrix W allows us to assign different weights to the observations. The expression to be minimised becomes

$$\underset{x}{\text{minimise}}(\mathbf{y} - A\mathbf{x})^T W(\mathbf{y} - A\mathbf{x}) \tag{A.11}$$

and its solution reads

$$\hat{x} = (A^T WA)^{-1} A^T Wy \tag{A.12}$$

For *weighted* least squares, $A^T W\hat{e} = 0$.

Example 2 – Averaging and least squares

Assume again the inconsistent system of the first example. Also assume that all observations have equal weight, i.e. $W = I_5$. The least squares solution follows from (A.12):

$$A^T WA = \begin{pmatrix}1 & 1 & 1 & 1 & 1\end{pmatrix}\begin{pmatrix}1\\1\\1\\1\\1\end{pmatrix} = 5$$

$$A^T Wy = \begin{pmatrix}1 & 1 & 1 & 1 & 1\end{pmatrix}\begin{pmatrix}9.99\\10.03\\9.98\\9.99\\10.01\end{pmatrix} = 50$$

and

$$\begin{aligned}\hat{x} &= (A^T WA)^{-1} A^T Wy \\ &= \frac{1}{5}\cdot 50 \\ &= 10\end{aligned}$$

This corresponds to the average of the previous example. Since we have derived that least squares is optimal in that it minimises the sum of the errors squared, $\sum e^2$, it follows that averaging is optimal in the same sense as well.

From now on we will assume that the vector y, which contains the numerical values of the observations, is actually a realisation of the *random* vector of observables $\underline{y}$. This vector of observables $\underline{y}$ will be written as the sum of a deterministic functional part Ax and a random residual part $\underline{e}$, which models the variability in the measurements

$$\underline{y} = Ax + \underline{e} \tag{A.13}$$

Here we will assume that the average variability in $\underline{e}$, or its *expectation*, is zero

$$E\{\underline{e}\} = 0 \tag{A.14}$$

where $E\{.\}$ denotes the *mathematical expectation* operator. The variability itself is characterised by the known *covariance matrix* Q_y

$$D\{\underline{e}\} = Q_y \tag{A.15}$$

where $D\{.\}$ is the *dispersion* operator, defined as $D\{.\} = E\{(. - E\{.\})(. - E\{.\})^T\}$. Using the propagation laws for means and covariances

$$E\{T\underline{u} + d\} = TE\{\underline{u}\} + d$$

$$D\{T\underline{u} + d\} = TD\{\underline{u}\}T^T$$

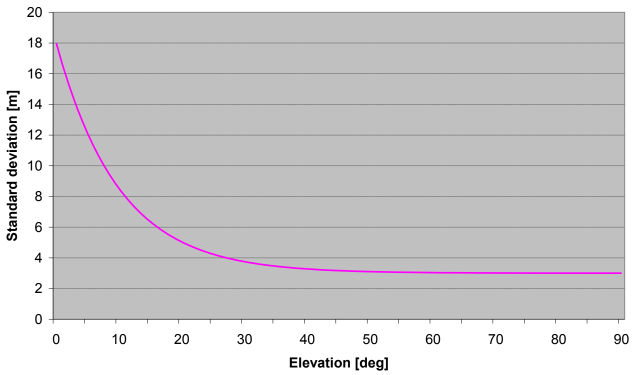
the measurement model can now be reformulated as

$$E\{\underline{y}\} = Ax \quad D\{\underline{y}\} = Q_y \tag{A.16}$$

Example 3 – Standard deviation of GPS code observations

The standard deviation of a GPS code observation is often expressed as a function of satellite elevation: the lower the satellite elevation, the less precise the observation. A possible expression for the standard deviation σ reads

$$\sigma(E) = A_0 + A_1 \exp(-\frac{E}{E_0}) \text{ [m]}$$



where E is satellite elevation, A_0 and A_1, both in meters, are non-negative constants and E_0, which has the same units as E, is a positive constant. Shown above are standard deviations as function of elevation for $A_0 = 3$, $A_1 = 15$ and $E_0 = 10$. Then for two satellites, one at 10° elevation, the other at 80°, the standard deviations are 3.0 m and 8.5 m, respectively. Their covariance matrix is given by

$$Q_y = \begin{pmatrix} 9 & 0 \\ 0 & 72.25 \end{pmatrix}$$

where it is assumed the observations are uncorrelated.

It can be proved that choosing $W = Q_y^{-1}$ in (A.12) results in estimators for the parameters that have minimal variance. Substituting $W = Q_y^{-1}$ into (A.12) and taking into account the stochastic nature of the observations finally results in the least squares solution used throughout this appendix

$$\underline{\hat{x}} = (A^T Q_y^{-1} A)^{-1} A^T Q_y^{-1} \underline{y} \tag{A.17}$$

$$\underline{\hat{y}} = A\underline{\hat{x}} = A(A^T Q_y^{-1} A)^{-1} A^T Q_y^{-1} \underline{y} = P_A \underline{y} \tag{A.18}$$

$$\underline{\hat{e}} = \underline{y} - \underline{\hat{y}} = (I - P_A)\underline{y} = P_A^{\perp} \underline{y} \tag{A.19}$$

The covariance matrix of $\underline{\hat{x}}$ follows from applying the covariance law to (A.17) as

$$Q_{\hat{x}} = (A^T Q_y^{-1} A)^{-1} \tag{A.20}$$

For the adjusted observations $\underline{\hat{y}}$ we get in a similar way

$$Q_{\hat{y}} = A Q_{\hat{x}} A^T \tag{A.21}$$

and for the least squares residuals $\underline{\hat{e}}$ finally

$$Q_{\hat{e}} = P_A^{\perp} Q_y (P_A^{\perp})^T = Q_y - Q_{\hat{y}} = P_A^{\perp} Q_y \tag{A.22}$$

So far we have assumed a linear relationship between observations and unknown parameters. If the relationship is nonlinear, i.e. $y = F(x)$, we can make it linear by expanding the function F into a Taylor's series around some approximate value x^0 for x and truncating after the second (linear) term

$$y = F(x^0) + \left.\frac{\partial F}{\partial x}\right|_{x^0} \Delta x \tag{A.23}$$

where the n-vector Δx is defined as $\Delta x = x - x^0$ and $\partial F / \partial x$ constitutes an $m \times n$ matrix, comparable to the design matrix A of the linear model (A.16). Introducing $\Delta y = y - F(x^0)$ and denoting $\partial F / \partial x$ by A we can now write for the linearised measurement model

$$E\{\Delta \underline{y}\} = A\Delta x \quad D\{\Delta \underline{y}\} = Q_y \tag{A.24}$$

Example 4 – Linearisation of the GPS code observation equation

The GPS code (or pseudo-range) observation PR can be considered as the sum of the geometric range R between receiver and satellite and the product of the speed of light c and the receiver clock bias $\delta_r t$. Expressed in receiver and satellite co-ordinates, denoted (x_r, y_r, z_r) and (x^s, y^s, z^s), respectively, the nonlinear observation equation reads

$$PR = \sqrt{(x_r - x^s)^2 + (y_r - y^s)^2 + (z_r - z^s)^2} + c\delta_r t$$

The satellite co-ordinates can be assumed known from the navigation message transmitted by the satellites. Denoting the approximate value of the receiver co-ordinates by (X_r^0, Y_r^0, Z_r^0), the linearised observation equation becomes

$$\begin{aligned}\delta PR &= PR - (R^0 + c\delta_r t^0) \\ &= \frac{\partial R}{\partial x_r}\Delta x_r + \frac{\partial R}{\partial y_r}\Delta y_r + \frac{\partial R}{\partial z_r}\Delta z_r + c\Delta\delta_r t \\ &= \frac{x_r^0 - x^s}{R^0}\Delta x_r + \frac{y_r^0 - y^s}{R^0}\Delta y_r + \frac{z_r^0 - z^s}{R^0}\Delta z_r + c\Delta\delta_r t\end{aligned}$$

where R^0 and $\delta_r t^0$ are the approximate values of the geometric distance and receiver clock bias for the first iteration.

The solution for Δx, $\Delta\hat{\underline{x}}$, follows from (A.17) and $\hat{\underline{x}}$ as $\hat{\underline{x}} = x^0 + \Delta\hat{\underline{x}}$. If x^0 is sufficiently close to x, then $\hat{\underline{x}}$ will be the final solution. However, if this is not the case, an iteration process is required. After each iteration, the estimator $\hat{\underline{x}}$ is used as the initial value for the next iteration. The procedure is repeated until the difference between subsequent solutions becomes negligible, i.e. if $\|\Delta\hat{\underline{x}}\|$ becomes negligible, also called the convergence criteria. The iteration process is shown schematically in Figure A.1.

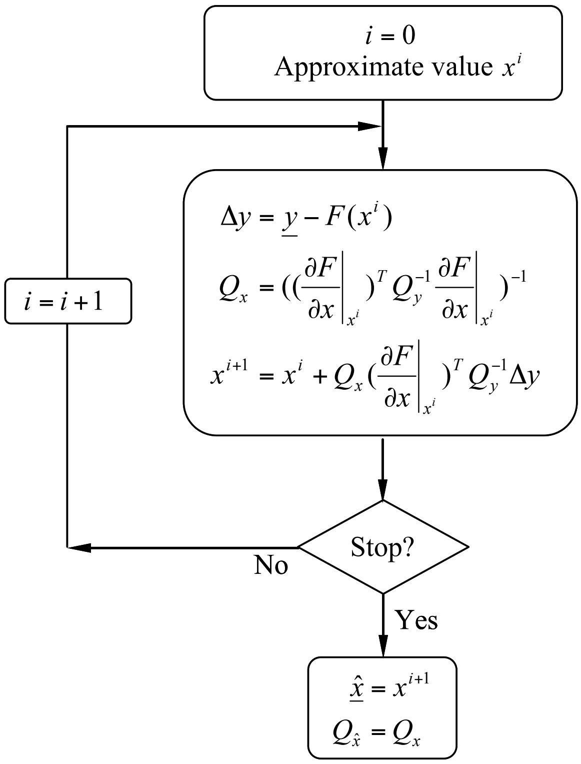


Figure A.1: Least square iteration

Example 5 – GPS code observation equation in geodetic co-ordinates

Consider the same pseudo-range PR as in the previous example, but now assume the receiver co-ordinates are required in geodetic latitude φ, longitude λ and height h instead of Cartesian x, y, z. The conversion from geodetic to Cartesian is given by

$$x = (N + h)\cos\varphi\cos\lambda$$
$$y = (N + h)\cos\varphi\sin\lambda$$
$$z = (N(1 - e^2) + h)\sin\varphi$$

with

$$N = \frac{a}{\sqrt{1 - e^2 \sin^2 \varphi}} \qquad e^2 = \frac{a^2 - b^2}{a^2} = (2 - f)f$$

a and b the reference ellipsoid's semi-major and semi-minor axis and f its inverse flattening.

The linearised observation equation now becomes

$$\delta PR = PR - (R^0 + c\delta_r t^0) = \frac{\partial R}{\partial \varphi_r}\Delta\varphi_r + \frac{\partial R}{\partial \lambda_r}\Delta\lambda_r + \frac{\partial R}{\partial h_r}\Delta h_r + c\Delta\delta_r t$$

The partial derivatives are given by

$$\frac{\partial R}{\partial \varphi_r} = \frac{\partial R}{\partial x_r}\frac{\partial x_r}{\partial \varphi_r} + \frac{\partial R}{\partial y_r}\frac{\partial y_r}{\partial \varphi_r} + \frac{\partial R}{\partial z_r}\frac{\partial z_r}{\partial \varphi_r}$$
$$\frac{\partial R}{\partial \lambda_r} = \frac{\partial R}{\partial x_r}\frac{\partial x_r}{\partial \lambda_r} + \frac{\partial R}{\partial y_r}\frac{\partial y_r}{\partial \lambda_r} + \frac{\partial R}{\partial z_r}\frac{\partial z_r}{\partial \lambda_r}$$
$$\frac{\partial R}{\partial h_r} = \frac{\partial R}{\partial x_r}\frac{\partial x_r}{\partial h_r} + \frac{\partial R}{\partial y_r}\frac{\partial y_r}{\partial h_r} + \frac{\partial R}{\partial z_r}\frac{\partial z_r}{\partial h_r}$$

In matrix notation these expressions become

$$\delta PR = \begin{pmatrix} \frac{\partial R}{\partial x_r} & \frac{\partial R}{\partial y_r} & \frac{\partial R}{\partial z_r} & c \end{pmatrix} \begin{pmatrix} \frac{\partial x_r}{\partial \varphi_r} & \frac{\partial x_r}{\partial \lambda_r} & \frac{\partial x_r}{\partial h_r} & 0 \\ \frac{\partial y_r}{\partial \varphi_r} & \frac{\partial y_r}{\partial \lambda_r} & \frac{\partial y_r}{\partial h_r} & 0 \\ \frac{\partial z_r}{\partial \varphi_r} & \frac{\partial z_r}{\partial \lambda_r} & \frac{\partial z_r}{\partial h_r} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \Delta\phi_r \\ \Delta\lambda_r \\ \Delta h_r \\ \Delta\delta_r t \end{pmatrix}$$

the first term on the right-hand side is already known from the previous example. Writing the second term as

$$\begin{pmatrix} \frac{\partial x_r}{\partial \varphi_r} & \frac{\partial x_r}{\partial \lambda_r} & \frac{\partial x_r}{\partial h_r} & 0 \\ \frac{\partial y_r}{\partial \varphi_r} & \frac{\partial y_r}{\partial \lambda_r} & \frac{\partial y_r}{\partial h_r} & 0 \\ \frac{\partial z_r}{\partial \varphi_r} & \frac{\partial z_r}{\partial \lambda_r} & \frac{\partial z_r}{\partial h_r} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} T & 0 \\ 0 & 1 \end{pmatrix}$$

the 3×3 matrix T follows from differentiating the expressions for the Cartesian co-ordinates, resulting in

$$T = T_1 T_2 = \begin{pmatrix} -\sin\varphi\cos\lambda & -\sin\lambda & \cos\varphi\cos\lambda \\ -\sin\varphi\sin\lambda & \cos\lambda & \cos\varphi\sin\lambda \\ \cos\varphi & 0 & \sin\varphi \end{pmatrix} \begin{pmatrix} (M+h)_0 & & \\ & (N+h)_0\cos\varphi_0 & \\ & & 1 \end{pmatrix}$$

where

$$M = \frac{b^2}{a(1 - e^2\sin^2\varphi)^{3/2}}$$

Instead of estimating the corrections $\Delta\varphi_r$ and $\Delta\lambda_r$, for numerical reasons often $(M+h)_0\Delta\varphi_r$ and $(N+h)_0\cos\varphi_0\Delta\lambda_r$ are estimated.

Example 6 – Height aiding

Although with the current GNSS constellation there usually are more than enough satellites in view to estimate a 3D position, sometimes this may not be the case. Height aiding may then be used to increase redundancy. Using the parameterisation in geodetic co-ordinates, as derived in the previous example, an additional observation of the height, h', is introduced

$$E\{h'\} = h \qquad D\{h'\} = \sigma_{h'}^2$$

Depending on the value for $\sigma_{h'}^2$, more or less weight can be assigned to this artificial observation. If $\sigma_{h'}^2$ is zero, the height is fixed, if it goes to infinity, it does not contribute to the solution. The linearised observation equation is given by

$$\delta h = h' - h_0 = \begin{pmatrix} 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} \Delta\varphi_r \\ \Delta\lambda_r \\ \Delta h_r \\ \Delta\delta_r t \end{pmatrix}$$

Note that if the height is fixed ($\sigma_{h'}^2 = 0$), $\Delta h = 0$.

Quality Control

The quality of an estimated set of parameters can be described in terms of precision and reliability. Precision, as described by the covariance matrix, expresses the estimated parameters' characteristics in propagating random errors. Often only the diagonal elements of this matrix are considered, since they describe the variances of the unknown parameters. However, such an approach does not take into account the correlation that exists between the parameters. Sometimes an even further simplification is made. If the observations all are assumed to have the same variance σ^2, then the covariance matrix of the estimated parameters (A.20) can be written as

$$Q_{\hat{x}} = \sigma^2 (A^T A)^{-1} \quad \text{(A.25)}$$

The dilution of precision (DOP) is defined as

$$\text{DOP} = \sqrt{\sum_{i=1}^{m} (A^T A)_{ii}^{-1}} \quad \text{(A.26)}$$

where the subscripts refer to the diagonal elements of the matrix $(A^T A)^{-1}$. In (A.26), all parameters are involved, but often only a subset of parameters is considered as well, see the example below. The DOP values indicate the influence of the geometry (through the design matrix *A*) on the precision of the parameters to be estimated. It should be stressed that DOP values themselves are not a good estimate of precision.

Example 7 – Dilution of precision (DOP) values

For GPS positioning using pseudo-range observations, parameters to be estimated consist of receiver co-ordinates (x, y, z) or (φ, λ, h) and the product of the speed of light c and the receiver clock bias $\delta_r t$. Some of the most commonly used DOP parameters are

GDOP (Geometric DOP): $\text{GDOP} = \sqrt{\sigma_\varphi^2 + \sigma_\lambda^2 + \sigma_h^2 + \sigma_{c\delta_r t}^2} / \sigma$

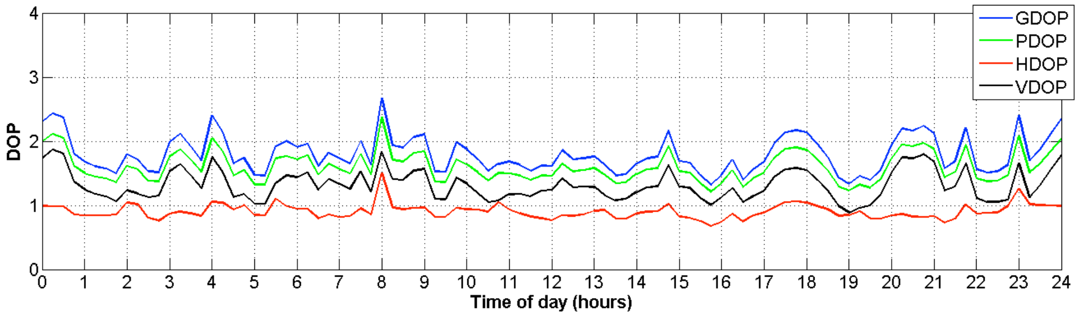
PDOP (Position DOP): $\text{PDOP} = \sqrt{\sigma_\varphi^2 + \sigma_\lambda^2 + \sigma_h^2} / \sigma$

HDOP (Horizontal DOP): $\text{HDOP} = \sqrt{\sigma_\varphi^2 + \sigma_\lambda^2} / \sigma$

VDOP (Vertical DOP): $\text{VDOP} = \sigma_h / \sigma$

TDOP (Time DOP): $\text{TDOP} = \sigma_{c\delta_r t} / \sigma$

For a location at 52° north and 4° east, DOP values for a particular day are given in the figure below.



Although modern satellite-based positioning systems provide positions in three dimensions, at sea one often is still mainly interested in horizontal (2D) positions and their corresponding precision. The area within which an estimated 2D position is likely to be is called *error ellipse* or *confidence region*. Let the covariance matrix Q of the horizontal position (x, y) be given by

$$Q = \begin{pmatrix} \sigma_x^2 & \sigma_{xy} \\ \sigma_{xy} & \sigma_y^2 \end{pmatrix}$$

then the semi-major axis a, the semi-minor axis b and the orientation angle γ (the angle between semi-major axis and horizontal co-ordinate axis) of the standard error ellipse follow from

$$\begin{aligned} a^2 &= \lambda_{\max} = \tfrac{1}{2}(\sigma_x^2 + \sigma_y^2) + \sqrt{\tfrac{1}{4}(\sigma_x^2 - \sigma_y^2)^2 + \sigma_{xy}^2} \\ b^2 &= \lambda_{\min} = \tfrac{1}{2}(\sigma_x^2 + \sigma_y^2) - \sqrt{\tfrac{1}{4}(\sigma_x^2 - \sigma_y^2)^2 + \sigma_{xy}^2} \\ \tan 2\gamma &= \frac{2\sigma_{xy}}{\sigma_x^2 - \sigma_y^2} \end{aligned} \tag{A.27}$$

where $\lambda_{\max}$ and $\lambda_{\min}$ are the largest and smallest Eigen values of Q, respectively. The probability that the horizontal position is within the standard error ellipse is 39.9%. To increase this probability to 95%, the ellipse should be scaled by a factor 2.448, see also Table A.1.

Instead of the error ellipse, which is defined by three parameters, in practice it is easier to use scalar quantities. The *distance root mean squared* (DRMS) is a single number which expresses 2D precision. It is defined as

$$\text{DRMS} = \sqrt{\sigma_x^2 + \sigma_y^2} \tag{A.28}$$

DRMS is also known as *mean squared position error* (MSPE), *radial error* or *root sum squared* (RSS). The probability of being within the circle with radius DRMS varies depending on the ratio between σ_x and σ_y. If both are the same, the probability is 63.2%, if their ratio is equal to 10, it is 68.2%. In practice, often 2DRMS (2×DRMS) is used, which corresponds to a probability between 95.4 and 98.2%

The *circular error probable* (CEP) is the radius of the 50% probability circle and is defined as

$$\text{CEP} \approx 0.589 \cdot (\sigma_x + \sigma_y) \tag{A.29}$$

This expression is valid for the range $0.2 \le \sigma_x / \sigma_y \le 1$, where it is assumed that $\sigma_x \le \sigma_y$.

Example 8 – Precision measures

Consider the covariance matrix

$$Q = \begin{pmatrix} \sigma_x^2 & \sigma_{xy} \\ \sigma_{xy} & \sigma_y^2 \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix}$$

From this matrix, the following quantities can be derived

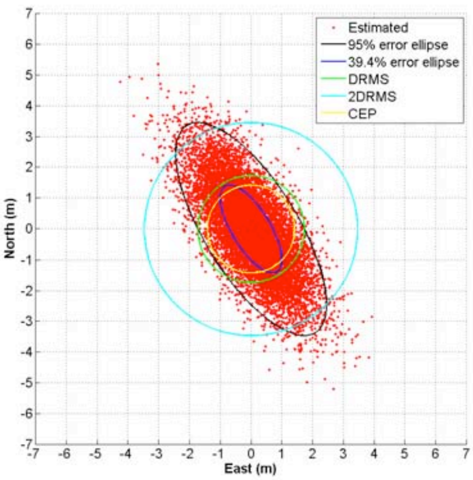
Semi-major axis of standard ellipse 1.6

Semi-minor axis of standard ellipse 0.6

DRMS 1.7

CEP 1.4

Standard error ellipse, 95% error ellipse, DRMS, 2DRMS and CEP are shown in the figure below, together with a large number of estimated positions, generated from normally distributed observations.



In three dimensions, the standard error ellipse becomes an ellipsoid, with axes given by the three Eigen values of the 3×3 covariance matrix. For the three dimensional case, the probability that the position is within the standard error ellipsoid is 19.9%. Expanding the axes by a common scale factor will result in an increased probability. Some probabilities (or confidence levels) and scale factors for one, two and three dimensions are given in Table A.1.

	Scale Factor		
Confidence Level [%]	**1D**	**2D**	**3D**
19.9			1.000
39.4		1.000	
68.3	1.000		
90	1.647	2.146	2.500
95	1.960	2.448	2.796
99	2.576	3.035	3.368
99.9	3.290	3.717	4.035
99.99	3.890	4.292	4.609

Table A.1: 1D, 2D and 3D scale factors.

The *mean radial square error* (MRSE) is a single number which expresses 3D precision and is defined as

$$\mathrm{MRSE} \approx \sqrt{\sigma_x^2 + \sigma_y^2 + \sigma_z^2} \tag{A.30}$$

where x, y, z are the three components of position. The probability of being in the sphere with radius MRSE is 61%. *Spherical error probable* (SEP) is the 3D equivalent of the 2D CEP. It is the so called 50% probability sphere, defined as

$$\mathrm{SEP} \approx 0.51 \cdot (\sigma_x + \sigma_y + \sigma_z) \tag{A.31}$$

Internal reliability describes the ability of the redundant observations to detect and identify specific model errors (or biases). *External reliability* expresses the influence of undetected model errors on the parameters of interest, i.e. co-ordinates.

The *null hypothesis* H_0 describes the case model errors are absent. The *alternative hypothesis* H_a considered here assumes there are model errors. Here we assume these model errors consist of one or more biases in the observations. These two hypotheses are defined as

$$H_0 : E\{\underline{y}\} = Ax \qquad D\{\underline{y}\} = Q_y \tag{A.32}$$

$$H_a : E\{\underline{y}\} = Ax + C\nabla \qquad D\{\underline{y}\} = Q_y \tag{A.33}$$

where C is a known $m \times b$ matrix, which specifies the type of model errors, and ∇ a b-vector containing the bias parameters. Note that $b \leq m - n$. The least squares solution to (A.32) under the null hypothesis and its covariance matrix are given by (A.17) and (A.20).

Testing H_0 against H_a consists of three steps:

- *Detection*: An overall model test is performed to find out if unspecified model errors have occurred.
- *Identification*: If model errors are detected, their potential sources are identified by testing the original or nominal observation model (A.32) against models extended with bias parameters, such as (A.33).
- *Adaptation*: After the identification of the most likely source of the model error, the observation model is adapted to eliminate the influence of biases in the parameter vector.

In the detection step the test statistic for testing H_0 against H_a is given as

$$T = \frac{\underline{\hat{e}}^T Q_y^{-1} \underline{\hat{e}}}{m-n} \tag{A.34}$$

The above test statistic T will be referred to as *overall model test statistic* or *F-test statistic*. Under H_0 and H_a, T has the *F* distribution

$$\begin{aligned} H_0 &: T \sim F(m-n, \infty, 0) \\ H_a &: T \sim F(m-n, \infty, \lambda) \end{aligned} \tag{A.35}$$

where λ is the *non-centrality parameter*.

The test statistic T in (A.34) can also be considered as a statistical method of determining whether the assumed variance matrix Q_y is realistic. The test statistic can be used to scale the covariance matrix to a more realistic value

$$Q_{y,new} = \sigma_0^2 Q_y \tag{A.36}$$

where $\sigma_0^2 = T$ is called *unit variance*. What is important is the average value of the unit variance over a period of time. If this is significantly different from unity, the most likely cause is incorrectness of the stochastic model, i.e. an incorrect covariance matrix of the observations. Alternatively there may be an unmodelled bias in the data, i.e. an incorrect functional model.

Once biases have been detected, i.e. if the test statistic T exceeds a threshold, the sources of the possible errors have to be found in the identification step. In practice this is accomplished by testing a number of alternative hypotheses, each describing one model error at a time. The matrix *C* in (A.33) then reduces to an m-vector *c*.

In the identification step, the uniformly most powerful test statistic for testing H_0 against H_a is given as

$$w = \frac{c^T Q_y^{-1} \underline{\hat{e}}}{\sqrt{c^T Q_y^{-1} P_A^{\perp} c}} = \frac{c^T Q_y^{-1} \underline{\hat{e}}}{\sqrt{c^T Q_y^{-1} P_A^{\perp} Q_y Q_y^{-1} c}} = \frac{c^T Q_y^{-1} \underline{\hat{e}}}{\sqrt{c^T Q_y^{-1} Q_{\underline{\hat{e}}} Q_y^{-1} c}} \tag{A.37}$$

Expression (A.37) is known as *w-test statistic* or *slippage test statistic*. Under H_0 and H_a, the slippage test statistic is normally distributed

$$\begin{aligned} H_0 &: w \sim N(0,1) \\ H_a &: w \sim N(\sqrt{c^T Q_y^{-1} P_A^{\perp} c}\,\nabla, 1) \end{aligned} \tag{A.38}$$

Example 9 – Slippage test statistic for uncorrelated observations

Assume we have a set of uncorrelated observations and that we want to test for an outlier in the i-th observation. Then

$$c_i = \begin{pmatrix} 0 \ldots 0 & 1 & 0 \ldots 0 \end{pmatrix}^T$$

where the non-zero element is at the i-th position. Since the observations are uncorrelated, we get for the slippage test statistic

$$w_i = \frac{\underline{\hat{e}}_i}{\sigma_{\underline{\hat{e}}_i}}$$

The practical procedure to identify model errors is to determine the largest slippage test statistic (in absolute value), remove the corresponding observation and perform another least squares adjustment until the overall model test statistic is accepted. Once the largest slippage test statistic has been found, its likelihood needs to be tested. The likelihood of the identified model error can be tested by comparing the test statistic with the critical value $N_{0.5\alpha_0}(0,1)$, where α_0 is the level of significance. If the largest slippage test statistic in each cycle exceeds the critical value, i.e. if

$$|w| > N_{0.5\alpha_0}(0,1) \tag{A.39}$$

it is likely that a model error has been identified. If not, one should consider the set of alternative hypotheses. It should be noted again that the testing methodology only tests for one outlier at a time, which may result in false identification, especially in cases with poor internal reliability. In addition it may also be necessary to test for two or more outliers at a time to avoid these false identifications.

For the adaptation step, several alternatives exist. One way to adapt would be to simply discard the bad observations (as already done, actually, in the iterated identification step), another to extend the vector of unknowns by one or more additional parameters. This means that the columns of the matrix C of (A.33) consist of the c-vectors, corresponding to the identified alternative hypotheses and that the new measurement model under the null hypothesis becomes in fact model (A.33).

The non-centrality parameter λ is defined as

$$\lambda = \nabla^2 c^T Q_y^{-1} P_A^{\perp} c \tag{A.40}$$

This parameter can be computed once reference values are chosen for the level of significance α_0 (the probability of a type 1 error, i.e rejecting H_0 falsely), the detection power γ_0 (the probability of rejecting H_0 when H_a is true) and the number of degrees of freedom $b = m - n$. Statistical testing is based on the B-method, developed at Delft University of Technology. The B-method assumes that an error related to the non-centrality parameter $\lambda_0 = \lambda(\alpha_0, \gamma_0, 1)$ is detected with equal probability, i.e. γ_0, by all tests. In other words from $\lambda_0 = \lambda(\alpha_0, \gamma_0, 1) = \lambda(\alpha, \gamma_0, b)$, with $b > 1$, the level of significance α can be computed. This implies that a certain model error can be found with the same probability by both the overall and the slippage test. A consequence of this coupling is that α increases when the redundancy increases.

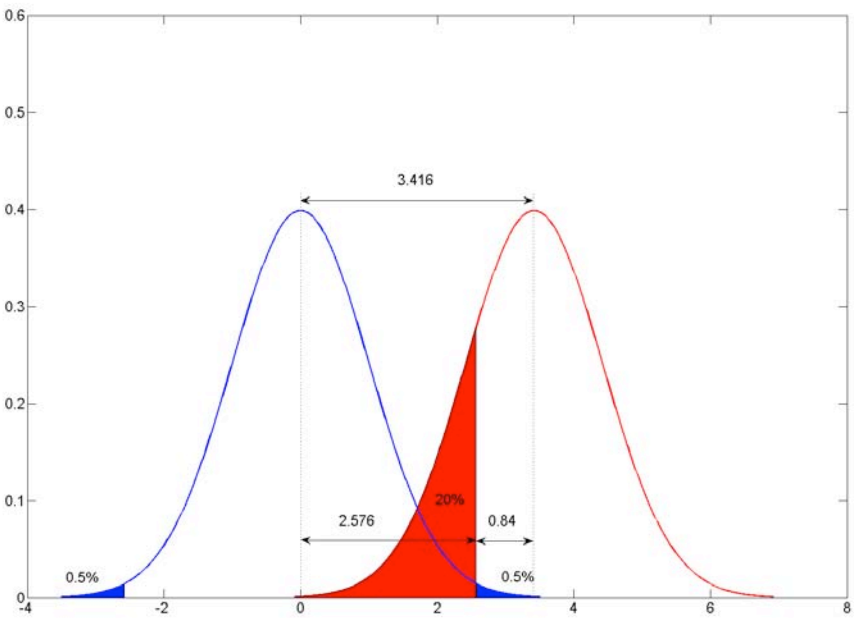


Figure A.2: Relationship between level of significance α_0 (1%) , power of test γ_0 (80%) and non-centrality parameter λ_0 (11.68 (3.416^2)), see also Table A.2.

Once the parameter $\lambda_0 = \lambda(\alpha_0, \gamma_0, 1)$ is known, the corresponding size of the bias that can just be detected follows from (A.40) as

$$|\nabla| = \sqrt{\frac{\lambda_0}{c^T Q_y^{-1} P_A^{\perp} c}} = \sqrt{\frac{\lambda_0}{c^T Q_y^{-1} Q_{\underline{\hat{e}}} Q_y^{-1} c}} \tag{A.41}$$

This is the (*observational*) *marginally detectable error* (MDE). For many practical applications, $\alpha_0 = 0.01$ and $\gamma_0 = 0.80$, resulting in a non-centrality parameter $\lambda_0 = 11.68$, see Figure A.2. Table A.2 gives some values for λ_0 as function of α_0 and γ_0. As can be seen from (A.31), the MDE not only depends on α_0 and γ_0, but also on the functional and stochastic model, through the design matrix A and the covariance matrix Q_y, and the alternative hypothesis considered, represented by the vector c. The alternative hypotheses may for example consist of outliers and cycle slips in GPS code and carrier observations, respectively.

α_0 = 0.1%		α_0 = 1%	
γ_0 [%]	λ_0	γ_0 [%]	λ_0
50	10.83	50	6.64
60	12.56	60	8.00
70	14.55	70	9.61
80	17.08	80	11.68
90	20.91	90	14.88

Table A.2: Non-centrality parameter λ_0 *as function of* α_0 *and* γ_0.

Example 10 – Observational MDE for uncorrelated data

Assume again a set of uncorrelated observations and that we want to compute the MDE for the i-th observation. Then, as in the previous example,

$$c_i = \left(0\ldots0 \quad 1 \quad 0\ldots0\right)^T$$

Since the observations are uncorrelated, we get for the MDE

$$|\nabla|_i = \frac{\sigma_{y_i}^2}{\sigma_{\underline{\hat{e}}_i}}\sqrt{\lambda_0}$$

The observational MDE is said to describe the internal reliability of a system. External reliability is defined as the influence of a bias with size equal to the MDE on the estimated parameters

$$\nabla\hat{x} = (A^T Q_y^{-1} A)^{-1} A^T Q_y^{-1} c\nabla \tag{A.42}$$

Since $\nabla\hat{x}$ is a vector and each alternative hypothesis results in such a vector, external reliability as described by (A.32) is in general hard to interpret. An easier to interpret alternative would be to compute the norm of the sub-vector of $\nabla\hat{x}$ which applies to the position parameters. The maximum norm will be referred to as *positional MDE.*

It should be noted that for the computation of MDEs, no actual data is required. They can already be computed in the design or planning stage of a survey and are important diagnostic tools to infer the strength with which observation models can be validated.

Example 11 – Internal and external reliability and the identification of biases

Pseudo-range observations from eight GPS satellites were collected. Standard deviation σ of the observations was assumed to be a function of satellite elevation

$$\sigma(E) = 3.0 + 15\exp(-\frac{E}{10}) \text{ [m]}$$

where E is satellite elevation in degrees. Level of confidence α_0 and power γ_0 where chosen as 1% and 80%, respectively, resulting in a critical value for the slippage test statistic of 2.576 and a non-centrality parameter $\lambda_0 = 11.68$ (see also Tables A.1 and A.2). The satellite constellation and the pseudo-range standard deviations are given in the table and figure below.

Observational and positional MDEs for this constellation are shown as well. Note that the largest observational MDE (satellite 31) does not correspond to the largest positional MDE (satellite 9). One of the reasons for this is that satellite 9 has a much larger weight than satellite 31, due to its higher elevation. However, satellite 5 has an even higher elevation, but although its observational MDE has about the same size as satellite 9's its positional MDE is much smaller. This shows that satellite-receiver geometry is also of importance.

A bias of 20m was added to the code observation of satellite 9. Shown are the absolute values of residuals and slippage test statistics.

Satellite	Azimuth (°)	Elevation (°)	σ (m)
2	107	17	5.8
4	70	19	5.1
5	264	73	3.0
9	132	49	3.1
12	15	87	3.0
14	283	46	3.1
29	197	16	6.1
31	302	5	11.9

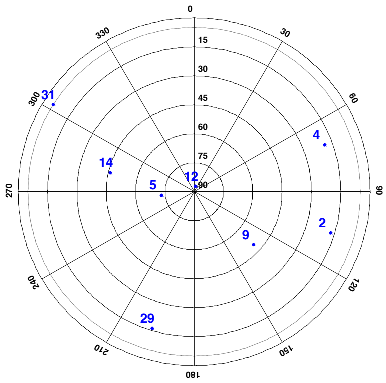
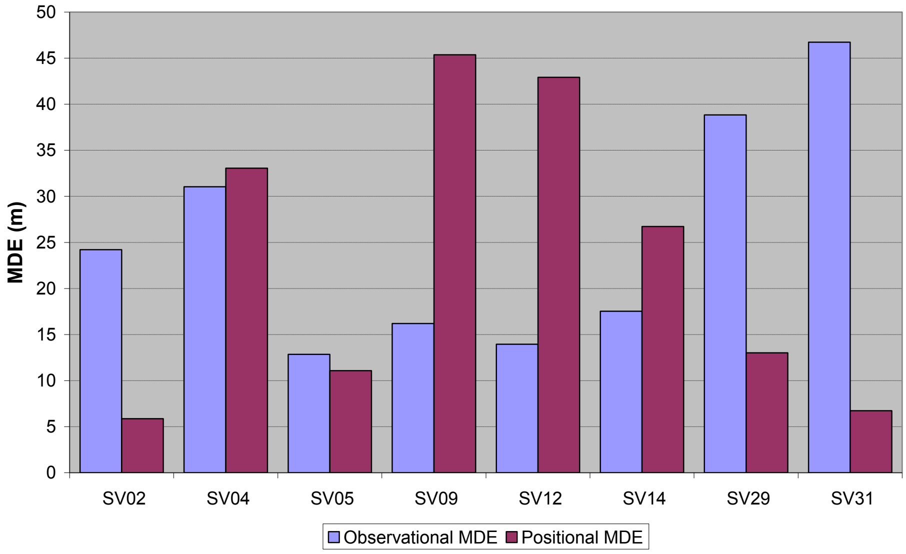
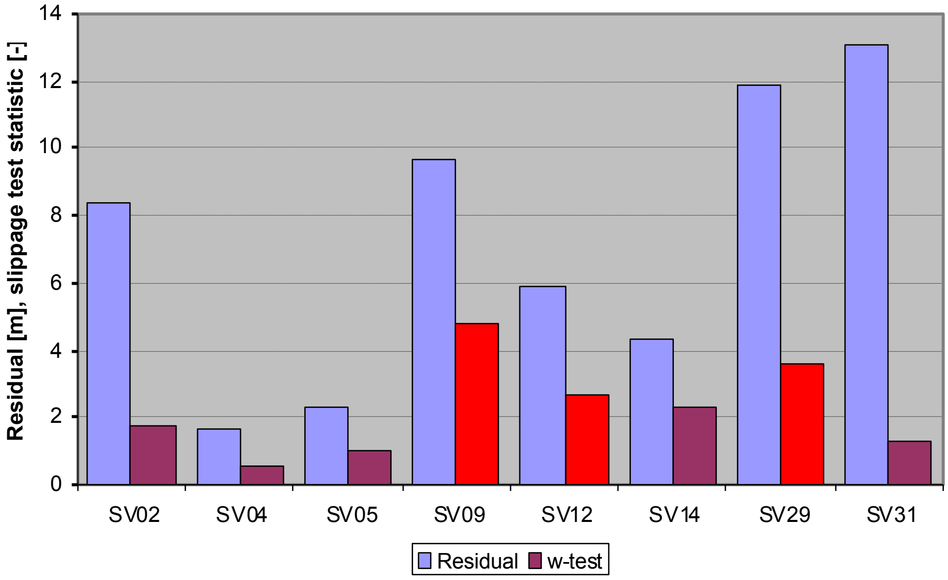


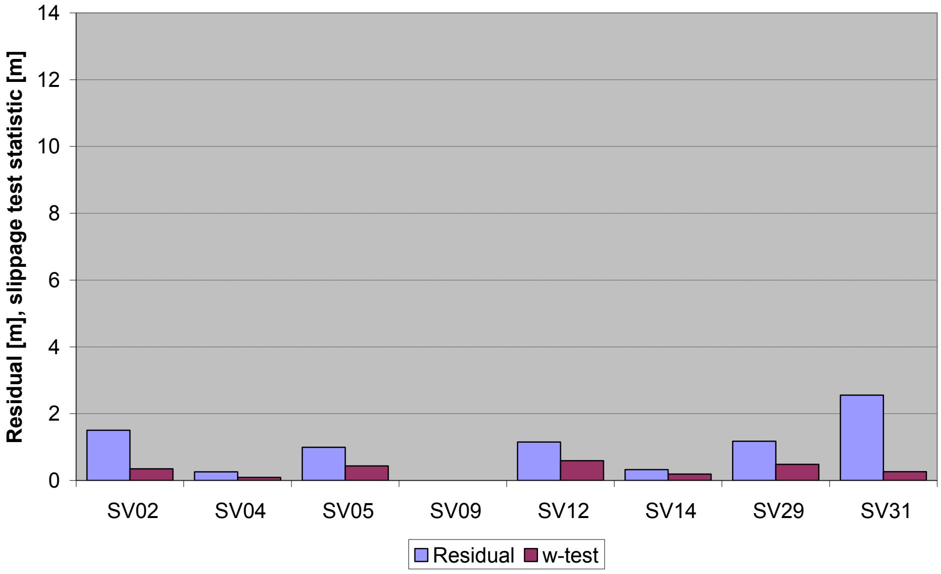
Table A.3: Satellite azimuth and elevation and observation standard deviation (left) and distribution of the satellites for the epoch considered

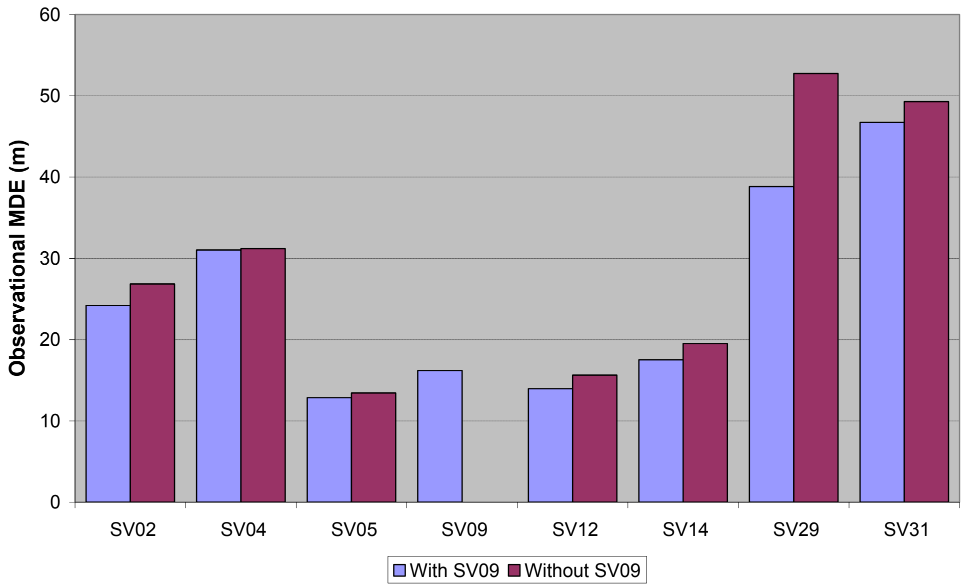
Note that the largest residual (satellite 31) does not correspond to the largest slippage test statistic (satellite 9). Also note that several slippage test values exceed the critical value of 2.576 (satellites 9, 12 and 29, indicated in red).

After removing satellite 9 (which has the largest slippage test statistic), all remaining residuals and slippage test values are much smaller; the test statistics are all below the critical value.


MDE (m)
50
45
40
35
30
25
20
15
10
5
0
SV02
SV04
SV05
SV09
SV12
SV14
SV29
SV31
Observational MDE
Positional MDE


Residual [m], slippage test statistic [-]
14
12
10
8
6
4
2
0
SV02
SV04
SV05
SV09
SV12
SV14
SV29
SV31
Residual
w-test



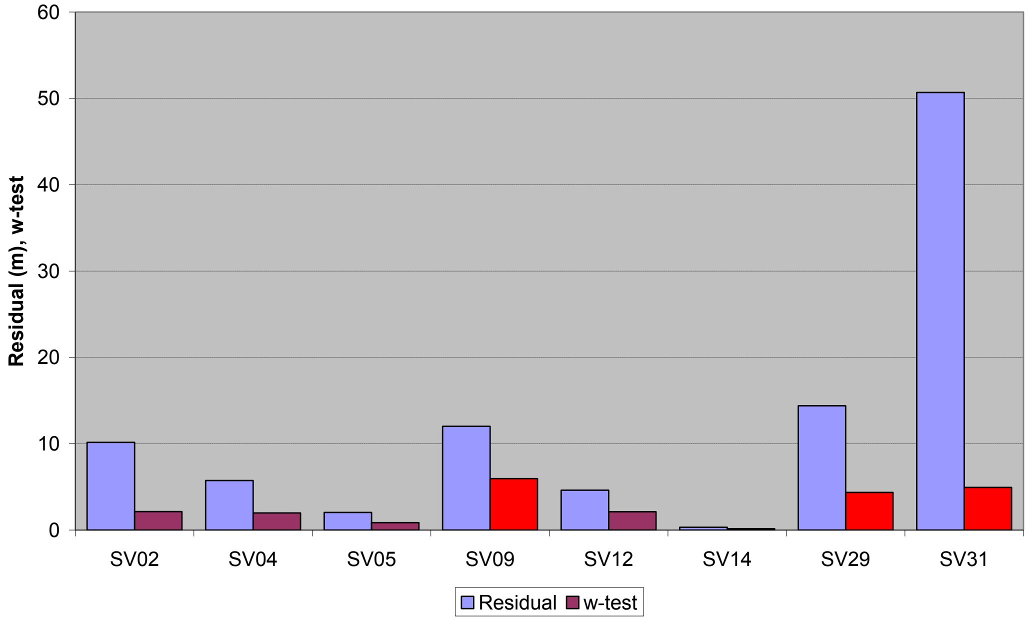


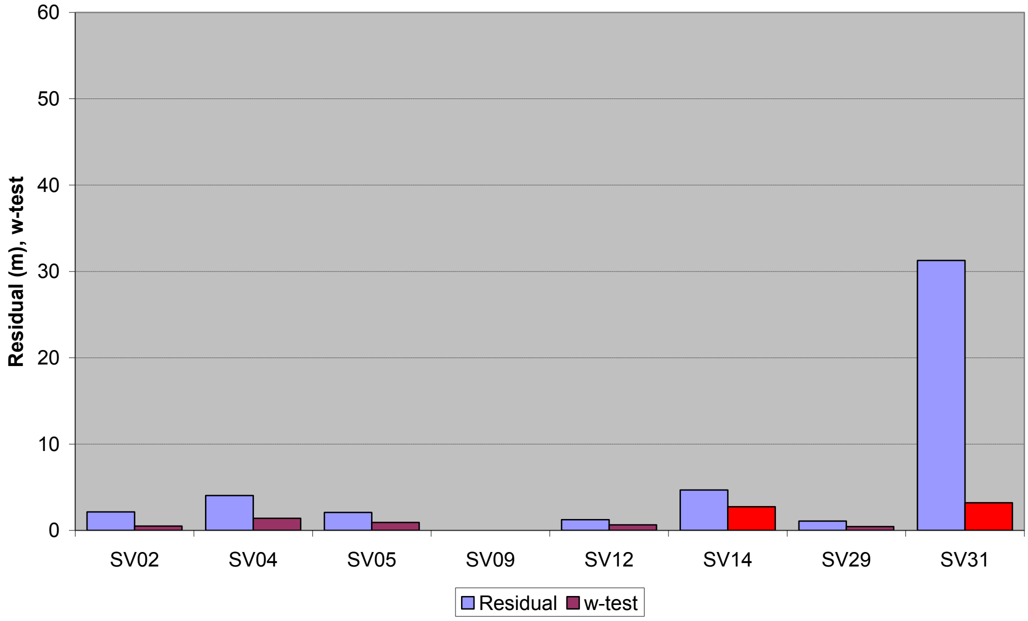
The four preceding figures, from top to bottom: Observational and positional MDEs for all observed satellites; residuals and local slippage test statistics after adding a bias of 20 m to the observation of satellite 9 (with test statistics exceeding the critical value shown in red); residuals and local slippage test statistics after removing satellite 9; observational MDEs with and without satellite 9.

It is interesting to compare observational MDEs with and without satellite 9. Since redundancy has decreased (satellite 9 has been removed from the solution), the MDEs for the seven remaining satellites have increased, i.e. it is more difficult to detect biases in the observations. Note that when satellite 9 was used in the solution, satellite 31 has the largest MDE. With satellite 9 removed, the largest MDE now corresponds to satellite 29.

Example 12 – Multiple outliers

The same data as in the previous example was used. Again, a bias of 20 m was added to the code observation of satellite 9. In addition, another bias of 50 m was added to the code observation of satellite 31. After the first adjustment, the w-test statistics indicated an outlier in the observation of satellite 9 (w-test statistics exceeding the critical value are indicated in red). After removing satellite 9, another adjustment was performed. After the second adjustment, the w-test statistics indicated an outlier in the observation of satellite 31. Note that the w-test statistic for satellite 14 now exceeds the critical value and that satellite 29's test statistic (which exceeded the critical value in the first adjustment) has dropped to 0.4. After removing satellite 31, the third adjustment did not indicate any outliers.

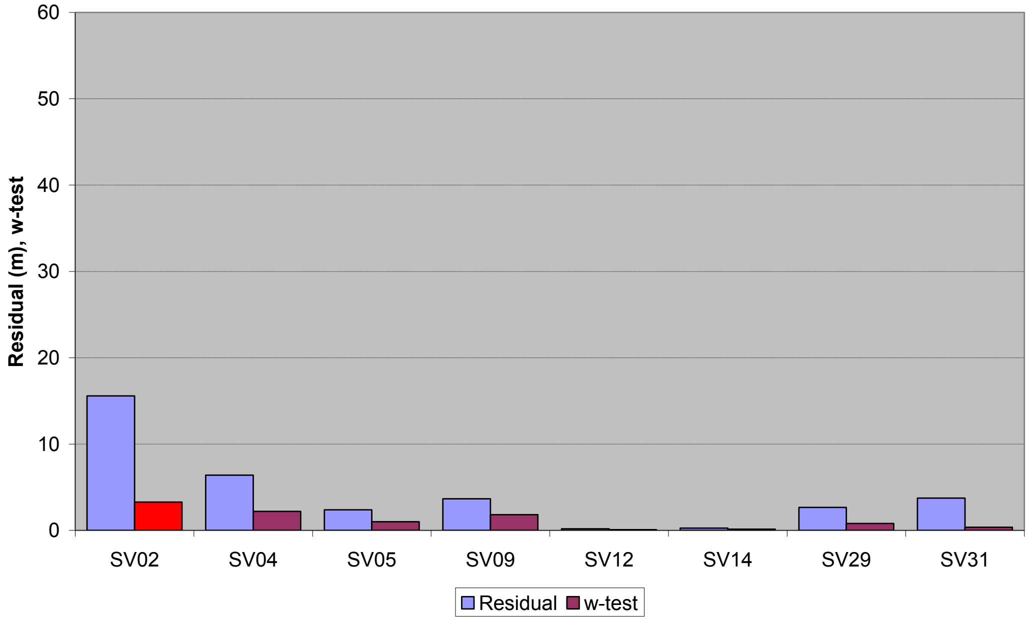




Top: residuals and local slippage test statistics after adding a bias of 20 m to the observation of satellite 9 and a bias of 50 m to the observation of satellite 31; bottom: residuals and local slippage test statistics after removing satellite 9. Slippage test statistics exceeding the critical value are shown in red.

Example 13 – Incorrect identification of biases

Again, the same data as in the previous examples was used and a bias of 20 m was added to the code observation of satellite 9. A bias of 50 m was added to the code observation of satellite 29 as well. After the first adjustment, the w-test statistics incorrectly indicated an outlier in the observation of satellite 2. After removing this satellite's observation and doing another adjustment, all observations (including those of satellites 9 and 29, containing the actual biases) were accepted.



Residuals and local slippage test statistics after adding a bias of 20 m to the observation of satellite 9 and a bias of 50m to the observation of satellite 29. Slippage test statistics exceeding the critical value are again shown in red.

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